

CURRENT TREND AND STUDIES IN MECHANICAL ENGINEERING



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Editor
Prof. Dr. İSMET SEZER





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A Thermal-Homeostatic Turbine Blade Inspired by Earth's Layered Heat Management and Skin Homeostasis: A Design Framework for Layered Protection, Regional Cooling, and Health Management

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ABSTRACT

This chapter presents a conceptual and computational framework that connects heat-management principles observed in two very different natural systems with gas-turbine blade design. From Earth's internal heat transport, it takes the ideas of layered thermal resistance and boundary-layer management; from human skin, it draws on barrier function, sensing, local response, and homeostasis. The aim is not to copy either natural system geometrically, but to transfer a set of functions that can be summarized as: layer, localize, sense, redistribute, and account for accumulated thermal load. A one-dimensional coating demonstration constructed at equal total thermal resistance shows that an FGM-like gradation can redistribute the temperature gradient through the thickness without artificially changing the total heat flux. In the three-region transient reduced-order model, V4 lowers the peak metal temperature from 1403.923 K to 1398.150 K relative to the V2 reference and reduces the normalized coolant integral by 5.01%. That nominal setting is nevertheless fragile under the prescribed parameter scatter: it satisfies the three-part success criterion in only 1 of 200 paired scenarios. V4-R, which combines temperature, filtered temperature rate, and cumulative thermal exposure, reaches a 95% joint pass fraction for the thermal criteria, but the mean normalized coolant saving falls to 0.44%. The economy-weighted V4-R2 produces an average saving of approximately 19.96% in an independent 200-scenario numerical test, yet fails the thermal criteria in every scenario. In a separate 800 s ablation demonstrator, the thermal-memory variable E reduces cumulative exposure by 29.28% relative to temperature-only feedback, while increasing normalized coolant use by 4.26%. The main outcome of the chapter is therefore a testable system hypothesis for subsequent three-dimensional conjugate heat-transfer (CHT), CFD, and experimental studies.

Keywords: turbine-blade cooling; bio-inspired design; thermal homeostasis; robustness under uncertainty; thermal barrier coating; structural health monitoring; coolant air.

1. INTRODUCTION

High turbine inlet temperature is a major design driver for cycle efficiency and power density in gas turbines. Because the hot-gas temperature can exceed the level that the metal can safely withstand for prolonged periods, internal cooling, film cooling, and thermal barrier coatings are used together [1,22]. This protection is not free. Cooling air extracted from the compressor imposes a parasitic penalty on cycle and thrust performance until it rejoins the main flow. The engineering problem is therefore not simply to cool the blade more

aggressively, but to preserve the required thermal margin with the smallest practical coolant budget.

The Earth analogy is not used here as an explanation for surface temperature. The feature of interest is the way a large temperature difference is managed through a moving internal medium, a conduction-dominated upper boundary layer, and a series of thermal resistances. Davies and Davies reported a total internal heat loss of 47 ± 2 TW and a global mean surface heat flux of about 92 mW m^{-2} [21]. This internal heat flux is not the dominant energy source for climate; the analogy is confined to the architecture of heat transport [11–13,21].

Human skin supplies a complementary function. The epidermal barrier protects the body from the external environment, while skin temperature participates in thermoregulatory feedback; local blood flow and sweating also change heat rejection regionally [16–19]. These features provide a useful abstraction for viewing the turbine blade not merely as a passively cooled component, but as a system that monitors its thermal state and redistributes limited cooling authority according to need.

The central question of this chapter is whether passive layered heat management inspired by the Earth and sensing-local-response homeostasis inspired by skin can be recast as a turbine-blade architecture that protects thermal safety within a defined reduced-order model and under prescribed hypothetical parameter scatter. The question is divided into three subproblems: whether regional allocation offers a pre-design advantage over equal allocation under a non-uniform heat load; whether a nominal minimum-coolant solution remains robust under uncertainty; and how temperature, temperature trend, and cumulative thermal exposure affect the safety-economy trade-off.

The novelty claim does not rest on film cooling, TBCs, ribs, optimization, sensing coatings, or structural health monitoring being individually new. Each already has a substantial literature: CHT-based and multi-objective turbine-cooling optimization [22], uncertainty analysis for film cooling [23], temperature-sensing TBCs with embedded thin-film thermocouples [24], self-healing TBC concepts [25], digital-twin-based life prediction for turbine-blade cracks [26], and broader SHM methods [27]. The contribution here is to place these otherwise separate research strands within one decision chain and explicitly test the difference between nominal design behavior and behavior under uncertainty.

Accordingly, the research gap is framed not as the absence of a single technology, but as a system-level integration problem coupled with a hierarchy of evidence. High-fidelity CHT optimization and physical sensing/SHM studies already exist [22–27]. This chapter operates at an earlier pre-design level, bringing layered protection, regional cooling authority, and thermal-memory feedback into the same decision loop. The numerical results are not validation data intended to replace established technologies; they form a reduced-order case study that identifies questions for later CFD/CHT and experimental investigation.

The chapter follows that hierarchy of evidence. It first examines the Earth and skin in terms of transferable functions, then reviews established turbine-blade cooling methods and the main design challenges. Section 4 converts the natural principles into a three-level thermal-homeostatic architecture and develops a thermal-resistance/allocation pre-model, the V4/V4-R/V4-R2 controller sequence, and a thermal-memory ablation test within the same framework. The final two sections discuss what the results do and do not support, and what is required for physical validation.

2. HEAT-MANAGEMENT PRINCIPLES FROM NATURAL SYSTEMS: EARTH AND SKIN

Fig. 1 is included not to map geological details onto turbine geometry, but to separate at a glance the distinct roles of the heat source, the moving transport region, and the conduction-dominated upper boundary. It provides the conceptual starting point for the layered-protection analogy used later in the chapter.

Layered heat-transfer mechanism of the Earth

The heat source is at the center; heat reaches the surface through conduction + convection. Turbine analogs are shown in teal.

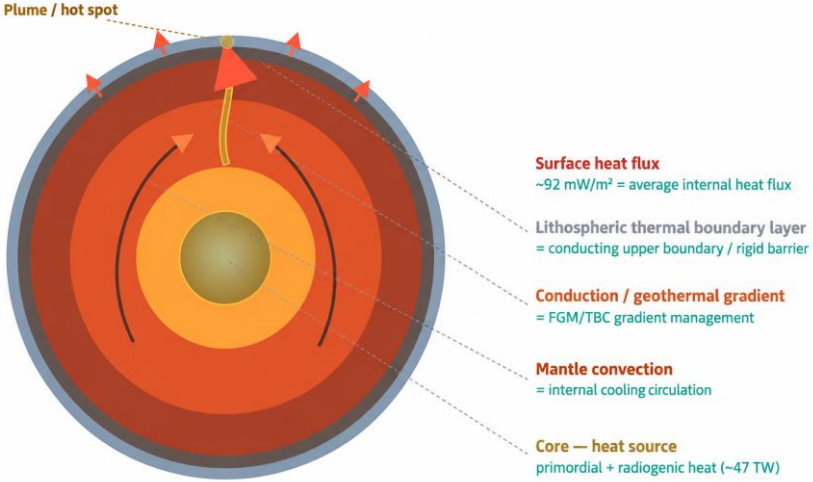


Fig. 1. Functional summary of the Earth's layered heat-transport mechanism: core and radiogenic heat sources, mantle convection, the lithospheric thermal boundary layer, and surface heat flux. The turbine correspondences are functional design analogies, not claims of physical scale similarity.

2.1 Earth's heat budget and layered structure

The global synthesis by Davies and Davies gives 47 ± 2 TW for the total heat transported from the Earth's interior to the surface; divided by surface area, this corresponds to approximately 92 mW m^{-2} [21]. Internal heat is supplied by a combination of secular cooling, radiogenic decay, and contributions from the core. Contemporary mantle models treat the system as mixed convection driven by both basal and internal heating [13]. These values do not imply that internal heat controls the Earth's surface climate. The analogy used in this chapter concerns only the way internal heat is transported through a layered structure.

2.2 Conduction, convection, and the thermal boundary layer

On geological time scales, the mantle behaves as an extremely viscous medium that deforms by solid-state creep. A substantial fraction of its heat is transported upward by convection [11–13]. It is therefore misleading to regard the mantle simply as a thick insulating blanket: internal circulation transports heat, while the near-surface lithospheric thermal boundary layer is largely conduction-controlled and limits the rate of transfer to the surface. The turbine

analogy likewise treats heat transport and thermal resistance as distinct design functions rather than as interchangeable mechanisms.

Whether a layer becomes convective is governed by the dimensionless Rayleigh number:

$$Ra = \frac{\rho g \alpha \Delta T d^3}{\mu \kappa} \quad (1)$$

Here, ρ is density, g is gravitational acceleration, α is the coefficient of thermal expansion, ΔT is the temperature difference across the layer, d is the layer thickness, μ is dynamic viscosity, and κ is thermal diffusivity. When Ra exceeds a critical value of approximately 10^3 , conduction alone is no longer sufficient and convection develops. In the Earth's mantle, Ra is very large, so convection is unavoidable [12].

2.3 Mantle convection: buoyancy, temperature-dependent viscosity, and a mobile upper boundary

Mantle circulation is driven by buoyancy arising from density differences. Hot material tends to rise because of thermal expansion, whereas material that reaches the upper regions, cools, and becomes denser participates in the return flow. Temperature-dependent viscosity strongly affects the form of this circulation. The present-day Earth's upper boundary is not a classical stagnant lid; plate tectonics produces a mobile-lid regime. The stagnant-lid concept remains useful for planetary comparisons, but it is not a direct description of modern Earth [11,12]. The transferable idea for turbine design is that the moving transport region and the conduction-dominated boundary layer perform different functions.

At high Ra , convection is time dependent and episodic: plumes rise intermittently and the system can behave chaotically. Models of secular cooling show that boundary layers evolve with time and help determine the heat flux [10].

2.4 Lithospheric boundary layer and geothermal gradient

The near-surface lithosphere is not completely stationary because of plate motion, yet relative to convective transport in the mantle it acts as an upper thermal boundary layer in which conduction is dominant. This layer is one of the structures that governs the rate at which internal heat reaches the surface and the associated temperature gradient. Heat also reaches the surface through

volcanism and hydrothermal circulation rather than by conduction alone [11–13]. From a design standpoint, the useful observation is that the total temperature drop need not occur within a single material.

The engineering lessons drawn from the Earth can be stated in four parts: distribute the temperature drop across layers; transport heat with a moving medium; allow transport properties to vary with operating state; and manage surface heat transfer through a distinct thermal boundary layer. None of these points implies physical equivalence. Their purpose is to provide a design filter that separates protection and transport functions within the blade.

2.5 Layered protection and thermal homeostasis in skin

Human skin is a multilayered, dynamic barrier between the environment and internal tissues. Epidermal homeostasis depends on balanced renewal among cell populations and also contributes to restoration of the barrier after injury [16]. The stratum corneum is not a single-property homogeneous layer. Using *in vivo* pH imaging at single-corneocyte resolution, Fukuda et al. identified three distinct pH zones within the stratum corneum [19]. The engineering point is that protective surface functions can be distributed through the thickness rather than concentrated in one uniform layer.

A second useful property of skin is feedback-regulated thermoregulation. Romanovsky argued that skin temperature is not merely a passive indicator of the environment; it carries feedback information within the thermoregulatory system and, in some regions, may also provide auxiliary feed-forward information [17]. Controlled human experiments by Wingo et al. showed that local skin temperature and changes in skin blood flow can independently modify the sweating response [18]. The principle transferred to the turbine blade is not a copy of the biological mechanism, but local and time-dependent adjustment of cooling in response to sensor information.

The Earth and skin do not solve the same physical problem. In the Earth example, the focus is transport of internal heat through layers and thermal boundary regions; in skin, barrier integrity, thermal sensing, and active local response are central. Their scales, materials, Reynolds/Rayleigh/Prandtl regimes, and time constants differ fundamentally from turbine flow. The chapter therefore transfers functions only; no direct geometric or dynamic similitude is assumed.

3. THERMAL MANAGEMENT IN TURBINE BLADES: ESTABLISHED TECHNOLOGIES AND DESIGN CHALLENGES

3.1 Temperature limits and the cycle cost of cooling

Turbine cooling air is extracted from the compressed flow at the compressor exit and routed through internal blade passages and film-cooling holes. This bleed flow carries penalties associated with pressure loss, compressor work, and the core-flow budget; thermal protection and coolant use must therefore be considered together [1,22]. The 'coolant use' reported in this chapter is not an actual bleed mass flow in kg s^{-1} . It is the time integral of the dimensionless/normalized cooling command used in the dynamic model.

3.2 Established cooling technologies

Three principal tools complement one another in gas-turbine blade cooling [1]: internal-channel cooling, film cooling, and thermal barrier coatings. None carries the entire thermal load by itself; the methods are combined according to blade geometry and mission profile.

In internal-channel cooling, cool air passes through multi-pass serpentine passages inside the blade. Ribs and turbulators disturb the near-wall thermal boundary layer and enhance convection; jet impingement at the leading edge provides high local heat removal. In the thin trailing-edge region, pin-fin arrays serve both as structural bridges and as mixing elements [1].

In film cooling, coolant discharged through surface holes forms a protective layer between the hot gas and the wall. Shaped or fan-shaped holes can spread the jet at the exit, reducing local momentum and the tendency to lift from the surface; at high blowing ratios they can therefore provide broader surface coverage than conventional cylindrical holes [14,15].

A thermal barrier coating (TBC) is a low-conductivity ceramic layer that adds thermal resistance between the metal and the hot gas. Yttria-stabilized zirconia (YSZ)-based coatings are among the most widely used systems in industry. Their protective thermal function and durability limits are well documented [1,20]. Functional grading provides an additional design route by smoothing abrupt material transitions and modifying the distribution of thermomechanical stress [3,4].

3.3 Mechanisms that complicate the design

(a) The heat load is non-uniform. The leading edge, pressure/suction surfaces, and trailing edge experience different local heat-transfer conditions; practical blade-cooling systems are therefore designed around regional requirements [1,22]. The three-region ROM used here represents only the pre-design consequence of this heterogeneity.

(b) Film-cooling lift-off. Film attachment depends on the combined effects of blowing ratio M , momentum-flux ratio I , density ratio, injection angle, hole geometry, freestream turbulence, and pressure gradient. The relation $I = M^2/DR$ is one of the basic indicators used to track jet penetration and lift-off tendency, but it is not the only determining parameter. As I increases, the counter-rotating vortex pair and jet penetration can strengthen and, under some conditions, lift the coolant film from the wall and reduce adiabatic effectiveness [15].

(c) The η - h trade-off (NHFR). The coolant film simultaneously reduces the driving temperature seen by the wall (benefit, η) but can disturb the boundary layer and increase the heat-transfer coefficient (penalty, h). The net benefit is:

$$NHFR = 1 - \frac{h_f}{h_0} \left(1 - \frac{\eta}{\phi} \right), \phi = \frac{T_\infty - T_w}{T_\infty - T_c} \quad (2)$$

η alone cannot determine whether film cooling is beneficial; η and h must be known separately [8].

(d) TBC delamination and low-cycle fatigue. Thermal-expansion mismatch between the ceramic coating and metallic substrate can increase interfacial stresses, particularly during start-stop cycles. FGM-TBC concepts seek to reduce stress concentrations by replacing an abrupt material transition with a graded one [3,4]. Coupled and thermomechanical analyses have shown that transient operating conditions alter blade temperature and thermal-stress fields [9].

(e) Unsteady film cooling. Rutledge's doctoral work and the subsequent experiments reported by Rutledge, King, and Rivir showed that unsteadiness in film-cooling jets may arise naturally in engine-like flow and may also be imposed deliberately for flow control [7,8]. Pulsation does not, by itself, provide a universal performance advantage. Drawing conclusions from effectiveness

alone can be misleading unless the time-varying adiabatic effectiveness and heat-transfer coefficient are considered together [8].

3.4 Bio-inspired cooling geometries: state of the literature

Bio-inspiration is not new to turbine cooling. Xing et al. numerically studied rhombus-patterned ribs inspired by shark-skin scales in steam-cooled internal channels and reported that decreasing rib height in the streamwise direction can improve overall thermal performance [5]. Liu et al. experimentally and numerically examined a turbine blade with tree-like fractal channels and showed that channel distribution has a marked effect on local cooling effectiveness [6]. These examples suggest that the engineering value lies less in copying a natural shape than in matching the flow path and surface area to the local thermal load.

3.5 Limits of the 'protective skin' analogy for a turbine blade

A turbine blade already contains a layered defense system in which the load-bearing superalloy, bond coat, thermally grown oxide (TGO), and ceramic TBC perform different functions. Darolia reviewed the thermal-protection role of this architecture together with damage mechanisms such as erosion, particle impact, and CMAS attack [20]. A 'sensing coating' is likewise more than a speculative future concept: Zhao et al. experimentally demonstrated measurement of the temperature gradient within a TBC using embedded thin-film thermocouples [24]. The novelty claimed here is therefore not the first proposal of a temperature-sensing TBC.

The same restraint is required on the health-management side. Self-healing TBCs form a distinct research field [25]; digital-twin approaches have been developed to update turbine-blade crack-growth predictions using experimental data [26]; and a broad SHM literature covers vibration, temperature, acoustic emission, ultrasonics, and related techniques [27]. The research hypothesis behind the 'engineering skin' proposed in this chapter is to connect such health information with regional coolant allocation and thermal-memory control within the same high-level decision loop.

Read together, the literature also clarifies the limits of the functional transfer. Biomimetic ribs and fractal channels show that heat-transfer enhancement must be assessed alongside flow resistance and distribution cost [5,6]. Film-cooling studies demonstrate sensitivity to unsteadiness and uncertainty

[7,8,23]. The skin literature supports barrier homeostasis, thermal feedback, and local response [16–19]. TBC, sensing-TBC, self-healing, and SHM/digital-twin studies show that passive protection and condition monitoring have matured as separate research directions [20,24–27]. This body of evidence supports using the natural analogy as a way to generate design questions rather than as a geometric template.

4. THERMAL-HOMEOSTATIC BLADE ARCHITECTURE AND REDUCED-ORDER CASE STUDY

4.1 Functional similarity and the limits of analogy

When a nature-derived idea is transferred to engineering, the first question should not be 'what does it look like?' but 'what function does it perform?' The Earth and a turbine blade may evoke superficial similarities, yet their heat-flow directions, flow regimes, materials, and time scales are not the same. The filter used here asks three questions: what function is being addressed, what physical means could serve that function in a turbine blade, and where does the analogy cease to be valid?

The difference is especially clear in the direction of heat flow. In the Earth, internal heat moves toward a colder surface. In a turbine blade, incoming heat from the hot mainstream must be limited and then removed from the metal to the coolant. The same idea of 'layering' can be retained, but the location of the heat source and the engineering objective are reversed.

Table 1. Functional mapping between heat-transport functions in the Earth and their turbine-blade engineering counterparts, with the corresponding level of evidence.

Earth mechanism	Transferred function	Turbine counterpart	Evidence status
Geothermal gradient / layers	Distribute the temperature drop across layers	FGM / multilayer TBC	Established [3,4,20]
Hotspot / plume spatiality	Recognize that the heat load is non-uniform	Regional allocation; bionic/fractal channel	Available [6]

Earth mechanism	Transferred function	Turbine counterpart	Evidence status
Mantle convection	Transport heat with a moving medium and renew the boundary	Internal coolant circulation; rib/impingement heat-transfer enhancement	Established; functional analogy [1,5]
Lithosphere / conductive upper lid	Limit rapid surface heat transfer through thermal resistance	Solid TBC / coating stack	Established [20]
Temperature-dependent rheology	Allow transport behavior to vary with state	State-dependent/adaptive cooling command	Functional analogy; no direct physical equivalence

Most of the engineering counterparts listed in Table 1 are already established technologies. The purpose of the table is therefore not to claim a new mechanism in every row, but to separate the functions of transport, thermal resistance, local loading, and state-dependent response so that the engineering means assigned to each role become explicit.

4.1.1 From skin homeostasis to adaptive thermal protection

The skin analogy is built around four functions (Fig. 2). The layered structure of the epidermal barrier [16,19] is mapped to a multilayer TBC/environmental-barrier concept [20]; the thermoregulatory information carried by skin temperature [17] is mapped to high-temperature sensors and sensing TBCs [24]; and the way local skin temperature and blood flow modify sweating [18] is mapped to variable-flow regional cooling. Barrier integrity and repair suggest a future high-level health-management function that connects damage information with thermal control and maintenance decisions. Self-healing TBCs and turbine SHM/digital-twin approaches already provide precursor components for this last level [25–27].

These functions are distilled into a computable control abstraction termed the 'thermal-homeostatic blade.' The controller does not rely on instantaneous temperature alone; it combines local temperature T , filtered temperature rate dT/dt , and cumulative thermal exposure above a critical threshold. The current state, trend, and thermal history therefore enter the same decision vector. In simple terms, the loop measures the state, evaluates risk, adjusts regional cooling authority and the total command cap, and then measures again.

Human skin analogy: layered protection + sense → respond → balance

Skin = barrier + thermoregulation + homeostasis. Turbine counterparts are shown in teal.

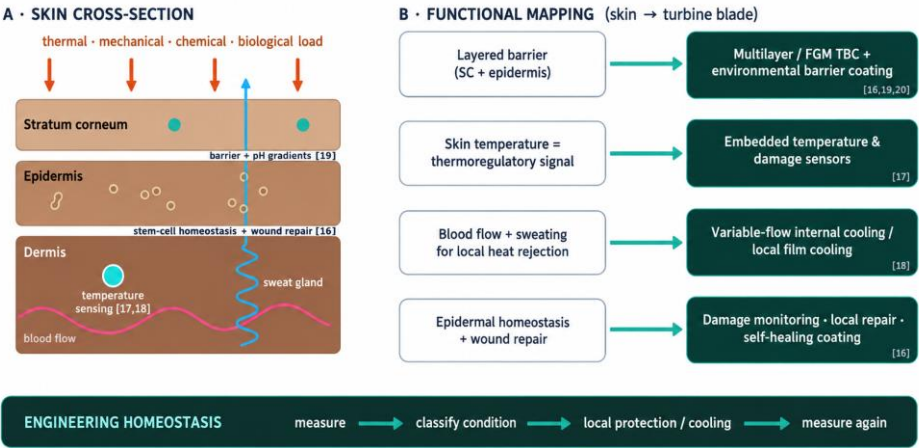


Fig. 2. Functional transfer of the skin analogy to a turbine blade: layered barrier, thermal sensing, local response, and feedback. The schematic is conceptual and does not contain experimental turbine data.

4.1.2 Three-level thermal-homeostatic blade architecture

The Earth and skin analogies are not collapsed into a single mechanism; instead, they are separated into three complementary engineering levels. Level I is passive layered protection. The TBC, bond coat, TGO, and load-bearing superalloy perform different thermal and mechanical roles, while an FGM approach can make abrupt material and thermal-expansion transitions more gradual [3,4,20].

Level II is active regional thermoregulation. The leading edge, mid-chord, and trailing edge do not experience the same thermal load. Cooling authority can therefore be varied according to local temperature, temperature trend, and cumulative thermal exposure rather than distributed uniformly and held fixed.

The quantitative contribution of this chapter is concentrated mainly at this level through the V4-R architecture and its uncertainty sweep.

Level III is thermal and structural health management. Individual examples of sensing TBCs, self-healing, crack prognostics, and SHM already exist in the literature [24–27]. In this chapter, these components are treated as potential inputs to a future supervisory architecture linking coating/crack health indicators with regional cooling commands and maintenance decisions. Table 2 summarizes the natural inspiration, turbine counterpart, and evidence status for all three levels.

Table 2. Transfer of Earth- and skin-inspired functions into the three-level thermal-homeostatic turbine-blade architecture and the evidence status of each level.

Architecture level	Natural inspiration	Turbine counterpart	Status in this chapter
I. Passive layered protection	Earth: gradient/boundary layer; skin: barrier	TBC/FGM, bond coat, TGO, load-bearing superalloy	Literature-based; one-dimensional equivalent-resistance demonstration [3,4,20]
II. Active regional thermoregulation	Skin: sensing, local blood flow/sweating, homeostasis	Sensor information + regional internal/film-cooling authority + V4-R	Quantified here with a ROM; no physical validation
III. Thermal/structural health management	Skin: barrier integrity and repair	Sensing TBC, damage map, SHM/digital twin, targeted maintenance/self-healing integration	Individual prior studies exist [24–27]; integrated loop is a conceptual future extension

4.2 Thermal resistance and regional allocation: pre-model

To connect the conceptual framework to a simple heat-transfer calculation, a series thermal-resistance chain is first constructed from the hot gas to the coolant. The following expression gives the equivalent resistance per unit area:

$$R_{\text{tot}} = \frac{1}{h_g} + \frac{t_{\text{TBC}}}{k_{\text{TBC}}} + \frac{t_m}{k_m} + \frac{1}{U_c} \quad (3)$$

Once the total resistance is defined, heat flux and metal temperature are obtained from the same energy balance. In the second expression, the

parentheses make explicit that the coolant-side and metal internal resistances are multiplied by the heat flux together:

$$q'' = \frac{T_g - T_c}{R_{\text{tot}}}, \quad T_m = T_c + q''(R_c + R_m) \quad (4)$$

Baseline correlations [1] are the Dittus-Boelter relation for a smooth channel and the Petukhov friction relation.

The gain-penalty comparison uses Webb's performance-evaluation approach [2]. Under the equal-pumping-power assumption, the thermal performance factor is:

$$TPF = \frac{Nu/Nu_0}{(f/f_0)^{1/3}}. \quad (5)$$

The exponent 1/3 follows from the scaling $P \propto fRe^3$.

Scaling laws. Internal cooling scales as $U_{\text{cool}} \propto m_{\text{dot}}^{0.8}$, reflecting the $Re^{0.8}$ dependence in the Dittus-Boelter correlation, while the pumping penalty scales as $P_{\text{pump}} \propto m_{\text{dot}}^3$ in the fully rough regime. This asymmetry makes the allocation problem meaningful.

The allocation problem is formulated to reduce the metal temperature of the hottest region under a fixed total-flow budget. In this subsection, m_{dot_i} is a generic pre-model variable representing physical flow rate; the m_i used in Section 4.3 is a dimensionless normalized control command. The two quantities should not be equated directly:

$$\min_{\{m_i\}} \max_i T_{\text{metal},i}(\dot{m}_i) \text{ s.t. } \sum_i \dot{m}_i = \dot{m}_{\text{tot}}. \quad (6)$$

Because $T_{\text{metal},i}$ decreases monotonically with flow rate, the optimum equalizes the temperatures of the active constraint regions.

4.2.1 One-dimensional demonstration of a layered thermal gradient

A one-dimensional conduction demonstration through the coating thickness is used to make the passive Earth-inspired idea of 'layering and gradient management' numerically visible. For a layer with spatially varying thermal conductivity, the conduction resistance per unit area is defined by the following integral:

$$R_{\text{cond}} = \int_0^L \frac{dx}{k(x)} \quad (7)$$

With fixed hot- and cold-surface temperatures, the total heat flux is obtained from this conduction resistance:

$$q'' = \frac{T_h - T_c}{R_{\text{cond}}} \quad (8)$$

The temperature at any position through the thickness can then be written in terms of the thermal resistance accumulated up to that point:

$$T(x) = T_h - q'' \int_0^x \frac{d\xi}{k(\xi)} \quad (9)$$

The demonstration uses a total coating thickness of $L = 300 \mu\text{m}$, a hot-surface temperature of 1500 K, and a cold-surface temperature of 1100 K. Homogeneous, three-layer, and continuously graded FGM-like profiles are scaled to the same total thermal resistance. The comparison therefore does not seek to change total heat flux, but to show how the same temperature drop is distributed through the thickness under the same heat flux.

Fig. 3 visualizes this controlled comparison. Because the three profiles share the same boundary temperatures and total thermal resistance, their total heat fluxes are also approximately equal. What changes is where the gradient is concentrated: the layered and continuous profiles distribute the temperature drop differently through the thickness. The figure does not establish coating-life or stress superiority; it illustrates only the geometric/material meaning of gradient management.

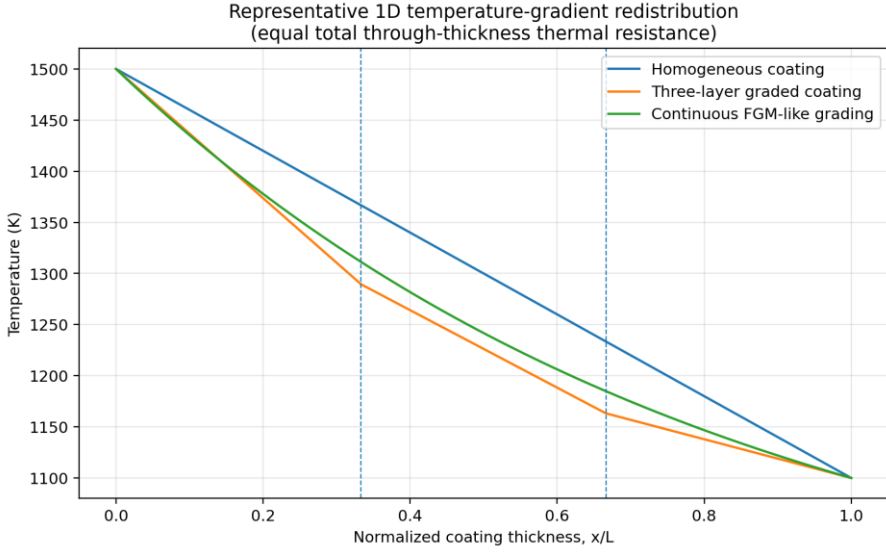


Fig. 3. Representative one-dimensional temperature distributions for homogeneous, three-layer, and continuously graded FGM-like coating profiles scaled to the same total thermal resistance. The comparison shows only how the gradient is redistributed; it is not evidence of coating-life or thermomechanical superiority.

Thermal stress, delamination life, or true FGM superiority cannot be inferred from the one-dimensional demonstration. Separate studies have shown that FGM-TBCs can alter interfacial stresses and thermal properties [3,4]; the present calculation represents only redistribution of the temperature profile under equal equivalent resistance.

4.3 V4-V4-R-V4-R2 development sequence and tests under uncertainty

4.3.1 Reduced-order model, parameters, and uncertainty definition

To make the V4-family results reproducible, the three-region thermal network is defined explicitly. Z1, Z2, and Z3 represent the leading edge, mid-chord, and trailing edge, respectively. Each region has its own effective thermal capacitance, exchanges energy convectively with the hot gas and coolant, and exchanges heat conductively with neighboring regions. The following equation gives the lumped energy balance:

In the equation, $G_{g,i} = h_{g,i} A_{g,i}$ and $G_{c,i} = h_{c,i} A_{c,i}$ are the convective conductances. The K_{ij} terms represent effective conduction between adjacent regions. The network does not replace a three-dimensional rotating CHT field; it is a

low-dimensional dynamic representation consistent with conservation of energy.

$$G_i \dot{T}_i = G_{g,i}(T_{g,i} - T_i) - G_{c,i}(T_i - T_c) - \sum_{j \in \mathcal{N}_i} K_{ij}(T_i - T_j) \quad (10)$$

The effect of coolant flow on the coolant-side heat-transfer coefficient is scaled with a Dittus-Boelter-type exponent of 0.8. The relation does not resolve channel or film-hole aerodynamics, plenum dynamics, or pressure loss; it represents monotonic cooling authority within the model:

$$h_{c,i} = h_{c,\text{ref},i} \left(\frac{m_i}{m_{\text{ref},i}} \right)^{0.8} \quad (11)$$

Cumulative thermal exposure collects, in a single memory variable, the magnitude and duration by which metal temperature exceeds $T_{\text{lim}} = 1350$ K:

$$E_i(t) = \int_0^t \max[T_i(\tau) - T_{\text{lim}}, 0] d\tau \quad (12)$$

To reduce sensitivity to noise in a direct numerical derivative, temperature rate is represented by a first-order filter state z_i . Equation (13) defines z_i explicitly as the filtered dT_i/dt signal:

$$\tau_d \dot{z}_i = \dot{T}_i - z_i \quad (13)$$

The regional risk signal combines positive temperature error, positive filtered heating rate, and normalized cumulative thermal exposure in the same expression:

$$r_i = K_{p,i} \max(e_i, 0) + K_{d,i} \max(z_i, 0) + K_{E,i} \frac{E_i}{E_s} \quad (14)$$

The local normalized cooling command combines the risk signal with the adaptive authority θ_i and an integral term. The min-max operation enforces local lower and upper command bounds; a total command cap is imposed separately at the supervisory level:

$$m_i = \min \left(m_{\text{max},i}, \max \left(m_{\text{min},i}, m_{\text{min},i} + \theta_i r_i + K_{I,i} I_i \right) \right) \quad (15)$$

The nominal V4 design seeks to reduce the total normalized cooling-command integral over a 120 s mission profile. Candidate solutions are first filtered by

the thermal feasibility conditions $T_{\text{peak}} \leq T_{\text{peak,V2}} + 2 \text{ K}$ and $E_T \leq 1.05 E_{T,V2}$; the 5% saving threshold is used separately in the scenario success criterion. The thermal constraint and the economic target are therefore not treated as if they were the same condition:

$$J_c = \int_0^{120} \sum_{i=1}^3 m_i(t) dt \rightarrow \min \quad (16)$$

The m_i values in Equation (16) are not physical bleed mass flows in kg s^{-1} ; they are normalized regional commands within the model. For the same reason, J_c is not an actual coolant mass or fuel-consumption metric. Reported percentage savings refer only to relative comparisons with V2 within the same ROM.

Table 3. Principal ROM parameters, controller settings, feasibility criteria, and uncertainty scenario used for the V4 family.

Group	Parameter / definition	Value / range	Comment
Thermal load	$T_{g,\text{base}} (Z1/Z2/Z3); T_c$	[1750, 1650, 1600] K; 850 K	Representative ROM input
Gas side	h_g	[2500, 1500, 1400] $\text{W m}^{-2} \text{K}^{-1}$	Nominal; not derived from physical testing
Effective areas	$A_g; A_c$	[0.010,0.012,0.009]; [0.009,0.011,0.008] m^2	Lumped model
Thermal storage	C_{th}	[450,520,380] J K^{-1}	Effective/representative
Inter-region conduction	$K_{12}; K_{23}$	7; 5 W K^{-1}	Effective/representative
Coolant scaling	$h_{c,\text{ref}}; m_{\text{ref}}$	Calibrated $h_{c,\text{ref}}; m_{\text{ref}}=[1,1,1]$	$h_c \propto m^{0.8}$; m normalized
V2 settings	$T_{\text{set}}; T_{\text{lim}}; m_{\text{min}}/m_{\text{max}}$	1300 K; 1350 K; [0.50,0.40,0.35]/[2.80,2.20,2.00]	PI-type regional reference
Mission profile	$t; T_0$	0–120 s; [1250,1230,1210] K	Stepwise thermal disturbance applied to the regions
V4 feasibility / success criterion	Thermal feasibility; economy threshold	$T_{\text{peak}} \leq T_{\text{peak,V2}} + 2 \text{ K};$ $E_T \leq 1.05 \times E_{T,V2}; \text{ saving} \geq 5\%$	First two conditions are hard thermal constraints for candidate screening; 5% saving is the scenario success threshold

Group	Parameter / definition	Value / range	Comment
V4 search	gain; cap; T_{set}	0.50–1.50; 4.86–5.98; 1290–1360 K + local refinement	Explicit grid + refinement
V4-R filter/memory	τ_d ; E_{scale} ; dT_{scale}	2 s; 1500 K·s; 6 K s ⁻¹	Control normalizations
V4-R authority/cap	$\theta_{\text{min}}-\theta_{\text{max}}$; cap-min/base/max	0.65–1.65; 4.95/5.20/5.98	Normalized cooling command
Uncertainty scenario	h_g , h_c , C_{th} , K, disturbance	$\pm 10\%$, $\pm 10\%$, $\pm 10\%$, $\pm 15\%$, $\pm 10\%$; independent uniform	Not a calibrated probability model
Sampling	V4/V4-R; R2 train/test	N=200, seed=42; $N_{\text{train}}=40$ seed=2026, $N_{\text{test}}=200$ seed=777	Empirical scenario pass fractions

The uncertainty sweep uses independent uniform multipliers of $\pm 10\%$ for h_g , h_c , and C_{th} ; $\pm 15\%$ for the common factor scaling K_{12} and K_{23} ; and $\pm 10\%$ for disturbance amplitude. These are not probability distributions statistically calibrated from an engine fleet. The percentages reported below are therefore empirical pass fractions within the prescribed scenario space rather than failure probabilities or certification-level reliability measures. Film-cooling studies have shown that operating and geometric uncertainties can materially affect effectiveness [23]; the present ROM carries that motivation into a lower-fidelity robustness screen.

The initial thermal-resistance/allocation calculation is a separate pre-model used to show that three regions exposed to different heat loads need not receive equal coolant shares. Table 4 lists the representative geometry-based gain-penalty inputs and the corresponding TPF values, while Table 5 compares equal allocation with the optimized normalized coolant allocation. The approximately 19% allocation difference and the reduction in peak temperature from 1413 K to 1372 K in this pre-model are kept separate from the subsequent V4/V4-R/V4-R2 performance metrics.

Table 4. Representative geometry-based gain-penalty inputs and calculated TPF values. The Nu/Nu_0 and f/f_0 values are not taken from a specific experimental or CFD data set.

Region / element	Nu/Nu_0 (representative)	f/f_0 (representative)	TPF
Z1 Impingement	4.5	16	1.79
Z2 45° rib	2.7	5.5	1.53
Z2 90° rib	2.2	9	1.06
Z3 Pin-fin	2.6	20	0.96

Table 5. Equal and optimized normalized coolant allocation in the representative pre-model.

	Z1 share	Z2 share	Z3 share	Total normal- ized cooling command
Equal allocation	33%	33%	33%	5.98
Optimal allocation	41%	29%	30%	4.86

In the pre-model, the ratio 4.86/5.98 corresponds to an approximately 18.7% reduction in total normalized cooling command for the same representative safety target. This result motivates regional allocation; the V4, V4-R, and V4-R2 results are obtained from a separate transient model with hard thermal constraints and uncertainty sweeps.

4.3.2 V4: minimum normalized-coolant selection under hard thermal constraints

V4 seeks to reduce the integrated normalized cooling command over 120 s while keeping the thermal behavior of the V2 reference within predefined tolerances. A coarse grid search followed by local refinement selects $gain = 1.3750$, $cap = 5.3900$, and $T_{set} = 1297$ K. V2 gives $T_{peak} = 1403.923$ K and a normalized coolant integral of 621.925; the corresponding V4 values are 1398.150 K and 590.783. Within the nominal model, the coolant integral is reduced by 5.01%, and the exposure ratio of 1.0309 remains below the defined limit of 1.05. These values are results of the calibrated ROM under the nominal scenario, not physical engine validation.

4.3.3 V4 robustness analysis: fragility of the nominal optimum

The sensitivity of the nominal V4 setting to parameter scatter is examined using 200 paired V2/V4 scenarios generated with the uniform multipliers in Table 3 (seed = 42). Success is defined by three simultaneous criteria: peak

difference ≤ 2 K, exposure ratio ≤ 1.05 , and normalized coolant saving $\geq 5\%$. Only 1 of 200 scenarios (0.5%) satisfies all three conditions. The mean saving is 4.42%, the worst saving is -2.52% , the worst peak difference is $+25.412$ K, and the worst exposure ratio is 1.396. The nominal minimum-coolant point is therefore clearly fragile within the prescribed model scatter.

4.3.4 V4-R: online homeostasis using state, trend, and thermal memory

V4-R replaces the fixed V4 settings with a two-layer control structure based on temperature, filtered dT/dt , cumulative thermal exposure E , and an online adaptive authority θ_i . The model does not estimate h_g and h_c directly online; their effects on metal-temperature response are handled through feedback. Across the same 200 uncertainty scenarios, the peak criterion is met in 190/200 cases (95%), the exposure criterion in 200/200 cases (100%), and both thermal criteria together in 190/200 cases (95%). Mean normalized coolant saving, however, is only 0.44% (median 1.36%; worst case -4.52%). V4-R is therefore not a controller that is simultaneously robust and economically successful; it converts much of the available economic margin into thermal safety margin. The Wilson 95% interval for 190/200 is approximately 91.0–97.3%, which is another reason not to present the 95% pass fraction as certification-level reliability.

4.3.5 V4-R2: independent test of the economy-thermal-safety conflict

V4-R2 is constructed to explore how far the economic target can be raised while drawing on the thermal margin of V4-R. A fixed 40-scenario training set is used to screen 162 hyperparameter combinations; thermal pass fraction is prioritized first, followed by mean normalized saving. The selected candidate is then evaluated on an independent 200-scenario numerical test set generated with a different random seed (seed = 777). In the nominal case, the economy-weighted R2 candidate produces a $+47.447$ K peak difference, an exposure ratio of 3.3823, and a 20.23% saving relative to V2. In the independent test, mean saving reaches 19.96%, while the peak, exposure, and joint thermal pass fractions all fall to 0%. The result shows that an aggressive saving objective can drift toward an unsafe solution when the thermal constraints do not generalize.

4.3.6 Integrating result: the nominal optimum is not the same as a robust solution under uncertainty

The V4 family reveals three distinct pre-design regimes. V4 provides about 5% normalized-coolant saving in the nominal case but is sensitive to parameter scatter. V4-R raises the thermal pass fraction to about 95% but does not meet the $\geq 5\%$ economic target. V4-R2 loses the thermal criteria in the independent test when the design is pushed too far toward economy. The main numerical result of the chapter is therefore not the selection of a single 'winning controller.' Rather, thermal safety and coolant use must be treated as a constrained, uncertainty-sensitive multi-objective problem. In a real engine design, the same problem would need to be reformulated in terms of CHT, pressure loss, physical bleed flow, and cycle performance [22,23].

Figs. 3–9 are used together to keep the different parts of the numerical chain separated by their level of evidence. Figs. 4, 5, and 7 are conceptual engineering schematics, whereas Figs. 3, 6, 8, and 9 present reduced-order or representative numerical results.

Fig. 4 is a conceptual cross-section showing how different cooling mechanisms can be matched to regional heat loads within the blade. Jet impingement is used at the leading edge, serpentine passages and ribs in the mid-chord region, and pin fins near the trailing edge, while the outer surface carries both a TBC and a coolant film. Its main message is to assign distinct functions to distinct regions rather than applying one uniform cooling strategy; the illustration should not be read as a numerical result.

Cooled Turbine Blade – Cross-Section

The coolant flow is graded according to the thermal load, and natural functions are implemented in an engineering design.

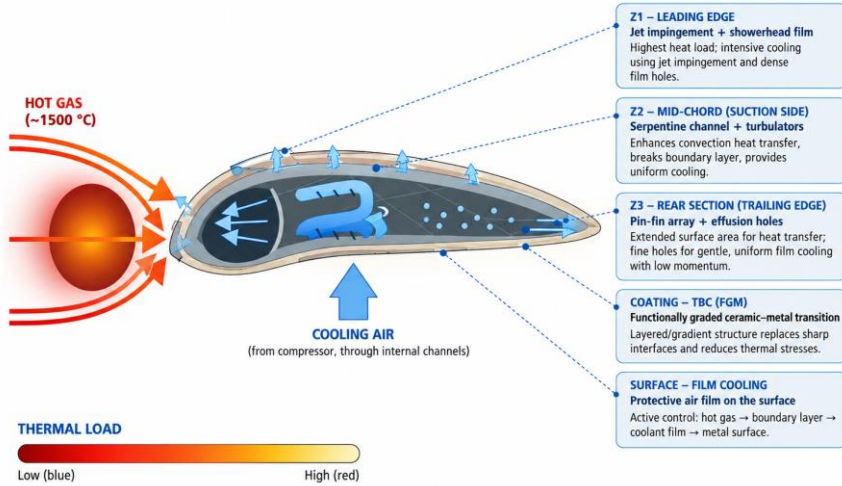


Fig. 4. Conceptual architecture combining regional cooling and layered protection within the same blade cross-section. The relationship to natural systems is functional only; the figure is not a CFD or experimental result.

Fig. 5 develops the same idea into a more detailed mechanism plate. The spanwise flow path, chordwise section, and near-wall mechanisms associated with jets, ribs, pin fins, and film cooling are shown together. The aim is not to copy a natural system geometrically, but to make the turbine-side physical counterpart of each transferred function explicit.

Internal Flow Path and Micro-Mechanisms

Coolant air is guided from the root to the tip. A multi-scale architecture and natural analogies are implemented at different levels.

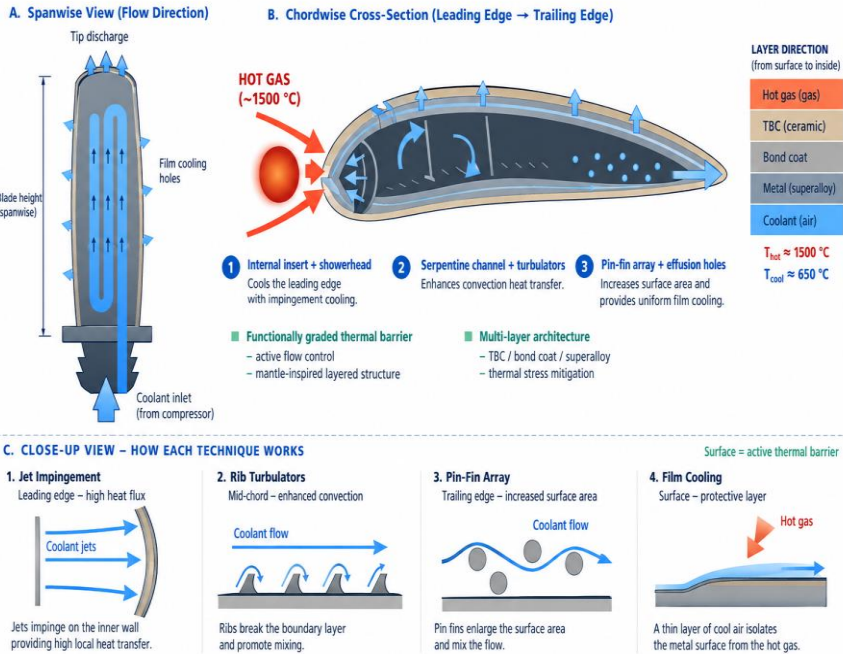


Fig. 5. Conceptual mechanism plate showing the internal flow path, chordwise section, jet impingement, ribs, pin fins, film cooling, and the wall thermal stack.

Fig. 6 summarizes the behavior of the pre-model in six panels: the wall-temperature profile, metal-temperature/flow relationship, regional temperature differences, cooling/pumping scaling, TPF comparison, and allocation result. The key observation is that regions exposed to different heat loads do not produce the same thermal response under equal coolant shares. Panel values are generated from representative pre-model inputs rather than from experimental or high-fidelity CFD data.

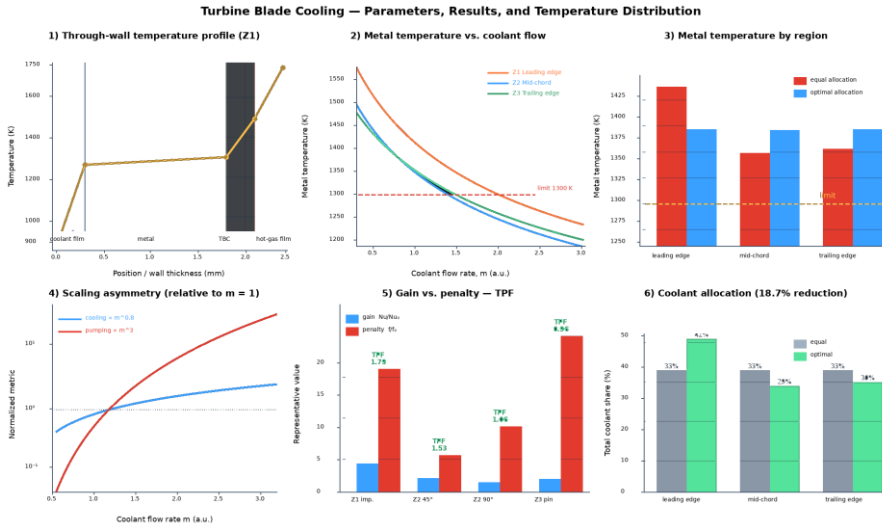


Fig. 6. Six-panel summary of representative pre-model results: temperature profile, metal-temperature/flow curves, regional temperatures, cooling/pumping scaling, TPF comparison, and allocation comparison. The values are not experimental or three-dimensional CFD data.

Fig. 7 schematically shows the counter-rotating vortex pair that can drive jet lift-off as momentum ratio increases in film cooling. Its role here is to emphasize that an adaptive controller cannot be built on the assumption that 'more coolant is always better.' Actual film performance depends jointly on blowing ratio, momentum ratio, density ratio, geometry, and turbulence [15].

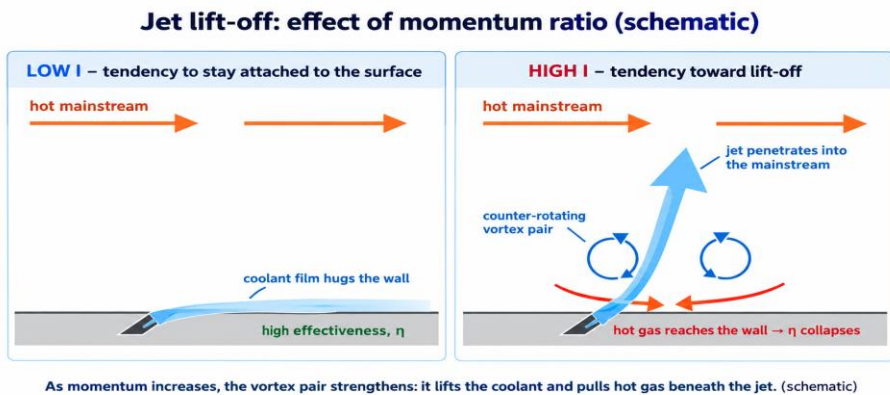


Fig. 7. Schematic representation of jet lift-off tendency and the counter-rotating vortex pair as momentum ratio increases in film cooling. The illustration is conceptual.

4.3.7 Minimal MATLAB ablation test of the homeostatic principle

A separate 800 s minimal demonstrator is used to isolate which information component carries the homeostatic benefit. This demonstrator does not use the same parameter set as the 120 s V4-family model; only relative differences among T , $T + \text{filtered } dT/dt$, and $T + \text{filtered } dT/dt + E$ are compared within the demonstrator itself. Cumulative thermal exposure E is calculated as the time integral of the portion of metal temperature above $T_{\text{lim,dem}} = 1300$ K; this threshold differs from the $T_{\text{lim}} = 1350$ K limit used in the V4 family. Peak temperature, thermal exposure, and normalized coolant use for the three control structures are reported together in Table 6.

Fig. 8 shows that the three regions develop different temperature histories under the same transient load cycle. Z1, the leading-edge region, carries the highest thermal load, while Z2 and Z3 remain cooler; the dashed 1300 K line is the exposure threshold used in this demonstrator. The figure therefore provides a direct visual argument for regional control: a single global temperature signal may not represent the local distribution of risk adequately.

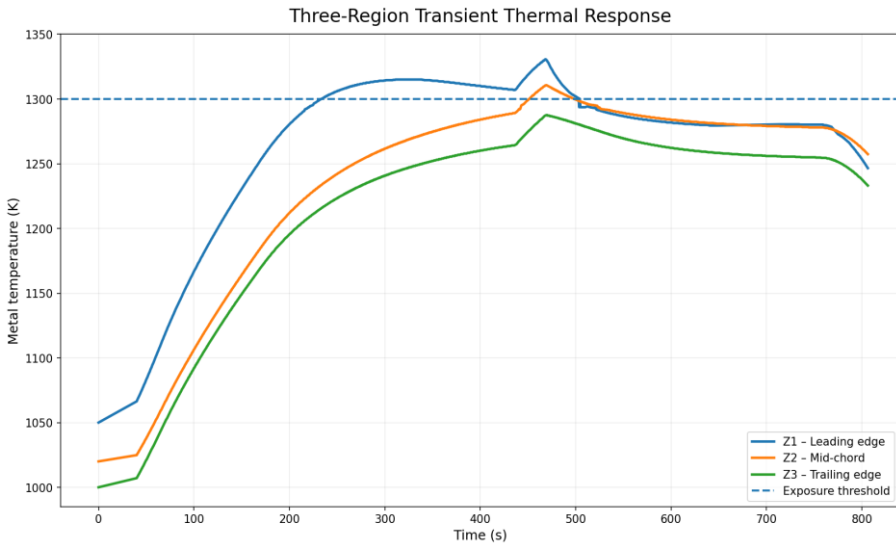


Fig. 8. Three-region transient metal-temperature response in the minimal homeostasis demonstrator. Z1 at the leading edge carries the highest thermal load, while Z2 and Z3 remain at lower temperatures.

Table 6. Minimal homeostasis ablation: relative effects of instantaneous state, trend, and thermal memory within the same demonstrator.

Control structure	Peak T (K)	Thermal exposure (K·s)	Normalized coolant use
T	1343.0	7205.3	1.000
T + filtered dT/dt	1341.8	7344.9	1.030
T + filtered dT/dt + E	1335.0	5095.3	1.043

The ablation result shows that temperature rate alone is not sufficient. Relative to the T-only reference, the T + filtered dT/dt structure lowers peak temperature by about 1.2 K but increases thermal exposure by 1.94% and normalized coolant use by 3.05%. When E is added, peak temperature falls from 1343.0 K to 1335.0 K and cumulative exposure decreases by 29.28%, while normalized coolant use rises by 4.26%. Adding E to the T + filtered dT/dt structure produces a further 30.63% reduction in exposure for only a 1.18% additional increase in coolant use. Within this model, the main source of homeostatic benefit is therefore the explicit retention of past thermal load as a memory variable in the decision loop.

Fig. 9 summarizes the ablation after normalization to the T-only reference. The structure with thermal memory substantially reduces cumulative exposure while increasing coolant cost more modestly. The plot should be read not as a literal reproduction of biological homeostasis, but as an engineering abstraction in which 'state + trend + past load' information manages a limited cooling budget.

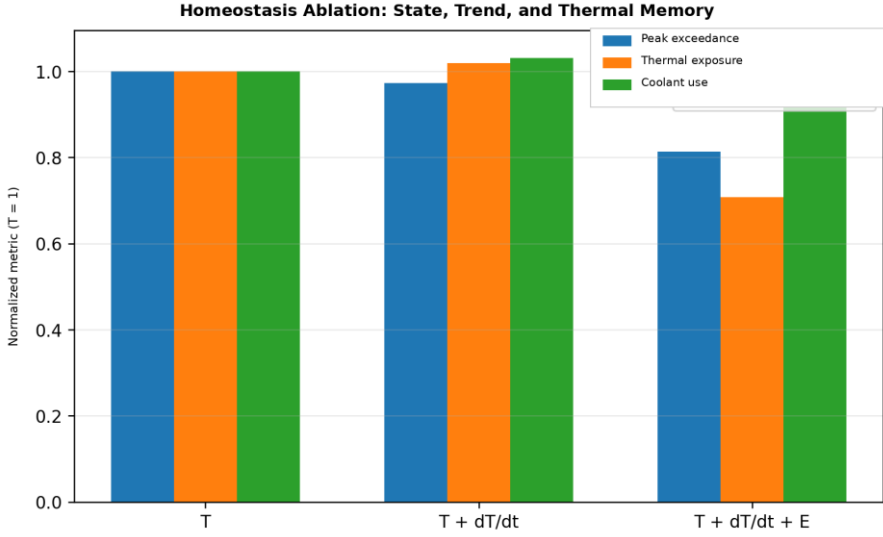


Fig. 9. Homeostasis ablation: normalized peak exceedance, cumulative thermal exposure, and coolant use relative to the T-only reference. The structure containing thermal memory E markedly reduces exposure.

A sample-count sensitivity check for the 800 s demonstrator shows that the mean peak temperature remains at 1336.5 K between 500 and 1000 samples; the 95th-percentile peak temperature changes from 1364.5 K to 1364.3 K, and the mean normalized coolant integral from 4113.7 to 4113.2. Cumulative exposure changes by about 3%. This convergence check assesses only the sample-count sensitivity of the minimal demonstrator.

4.3.8 Scale separation and validation pathway

The commonality among the Earth, skin, and a turbine blade exists only at the functional level. Mantle convection is viscous solid-state convection at high Rayleigh/Prandtl number and geological time scales. Skin involves biological circulation, metabolism, and sweating. A turbine blade operates at high Reynolds and Mach numbers with compressibility, rotation, turbulence, and intense external heat flux. No Reynolds-Rayleigh similitude or directly scalable geometry is therefore proposed. The next level of evidence is to calibrate the ROM parameters against three-dimensional CHT/CFD or thermal-rig data and then test the same control logic with realistic pressure losses, film/impingement interactions, and sensor/actuator dynamics [22,23].

5. INTEGRATED ASSESSMENT OF THE RESULTS: SAFETY, ECONOMY, AND HEALTH MANAGEMENT

Taken together, V4, V4-R, and V4-R2 make it clear that minimizing the normalized coolant integral alone is not a safe design criterion. V4 achieves a 5.01% saving in the nominal model but satisfies the three-part success criterion in only 1 of 200 prescribed uncertainty scenarios. V4-R raises the joint thermal pass count to 190/200, but the mean saving falls to 0.44%. Within the same ROM, nominal economy and thermal protection under uncertainty are therefore distinct objectives.

From a homeostatic-control perspective, the distinguishing feature of V4-R is the use of three time-related pieces of information. T represents the instantaneous thermal state, dT/dt its direction and rate of change, and cumulative thermal exposure the memory of past load. This state-trend-memory structure provides a richer risk signal than temperature alone and offers a generalizable control idea for early thermal-risk management.

The minimal ablation supports that interpretation quantitatively. Filtered dT/dt by itself does not reduce cumulative exposure; the clear improvement appears only when E is added to the control loop. Relative to the T -only reference, exposure falls by 29.28% while normalized coolant use rises by 4.26%. The idea of 'memory' therefore becomes a measurable control function within the same ROM rather than remaining only a biological analogy.

Health management remains a supervisory extension in the present version. Sensing TBCs [24], self-healing TBCs [25], digital-twin crack prognostics updated with experimental data [26], and several SHM methods [27] already exist as independent research directions. The future research path proposed here is to integrate such health indicators with regional thermal control and maintenance decisions in the same decision loop; the present ROM does not couple TBC delamination, CMAS/erosion, thermal stress, creep/fatigue life, or thermal-cycle count to its state equations. Table 7 summarizes the natural design principles, their turbine counterparts, and the level of evidence reached in this chapter.

Table 7. Design principles derived from the Earth-skin-turbine-blade analogy, their turbine counterparts, and the corresponding level of evidence.

Design principle	Natural source	Transferred function	Turbine-blade counterpart	Status in this chapter
Layer	Earth + skin	Distribute thermal load across a multilayer resistance/barrier	TBC/FGM + bond coat + load-bearing metal	One-dimensional equivalent-resistance demonstration + literature [3,4,20]
Localize	Earth hotspot/plume + local skin response	Recognize that the heat load is non-uniform	Z1-Z2-Z3 regional thermal design	Three-region MATLAB ROM
Sense	Skin thermoregulation	Measure instantaneous thermal state and its trend	T and filtered dT/dt sensor signals	V4-R / minimal ablation
Redistribute	Skin blood flow and sweating	Direct a limited transport budget according to need	Regional coolant allocation; internal/film-cooling authority	Pre-model + V4-R
Remember	Engineering abstraction derived from skin homeostasis	Convert past thermal load into a decision variable	Cumulative thermal exposure E	Minimal ablation: exposure -29.28%; normalized coolant +4.26%
Monitor health / repair	Skin barrier integrity and wound repair	Link protection decisions to damage state	Coating-health monitoring, targeted maintenance, future self-healing	Individual prior studies exist [24–27]; integrated thermal-health loop not validated

In this synthesis, the Earth analogy contributes the passive principle 'layer and manage the gradient,' whereas the skin analogy contributes the active principle 'sense, localize, redistribute, and remember.' Using both analogies is meaningful only insofar as it generates new and testable research questions rather than merely renaming existing cooling methods: How should the coating gradient be selected? Which region should receive limited coolant, and when? Which

risk indicator should be combined with thermal memory? At what level of evidence should health information enter the control loop?

6. LIMITS OF THE CONTRIBUTION AND VALIDATION ROADMAP

The Earth-skin mapping is a design framework, not a new heat-transfer mechanism. CHT optimization, film-cooling uncertainty analysis, sensing TBCs, self-healing, and SHM/digital-twin components already exist separately in the literature [22–27]. The contribution is not to replace those strands, but to connect layered protection, regional normalized cooling authority, state-trend-memory feedback, and prescribed uncertainty scenarios within one low-dimensional development chain.

The claim boundary is equally clear for the one-dimensional FGM demonstration. The homogeneous, three-layer, and continuously graded profiles are deliberately scaled to the same total equivalent thermal resistance; under the same boundary temperatures, nearly equal heat flux is therefore a natural consequence of the model setup. The calculation shows only how the temperature gradient is redistributed through the thickness. Interfacial stress, delamination life, or coating superiority require separate thermomechanical and experimental evidence [3,4,20].

Skin homeostasis and turbine cooling are not the same physical process. What is transferred is not biological material or geometry, but the functional sequence of sensing-feedback-local response-maintenance of balance. V4-R turns that abstraction into a measurable control architecture using T , filtered dT/dt , and cumulative thermal exposure.

1. Methodological contribution. The model equations, calibration approach, principal parameters, grid search, thermal feasibility conditions, and uncertainty ranges are stated explicitly. V4 uses a minimum normalized-coolant search under hard thermal constraints; V4-R combines state-trend-memory feedback with a paired-scenario sweep. V4-R2 serves as a negative control showing how a stronger economic emphasis can degrade thermal constraints in an independent test.

2. Physical-model limitation. The three-region network is a lumped model that is consistent with energy conservation, but h_g , h_c , A , C_{th} , and K_{ij} are not derived from a three-dimensional rotating-blade CHT field or from thermal-rig

measurements. The $h_c \propto m^{0.8}$ scaling is monotonic and does not resolve film momentum ratio and lift-off, compressible mixing, passage/hole geometry, plenum dynamics, or pressure loss. Absolute temperatures are therefore pre-design outputs rather than certification results.

3. Uncertainty and statistical limitation. The $\pm 10\%/\pm 15\%$ uniform ranges are not probability distributions calibrated from field data. The reported percentages are empirical scenario pass fractions. With $N = 200$, finite-sample uncertainty remains relevant, particularly for rates near 95%. The word 'robustness' in this chapter refers only to behavior within the prescribed model and scenario space; it does not imply a real-world failure probability [23].

4. Coolant and cycle limitation. The m_i and J_c quantities are normalized model commands. Actual bleed flow in kg s^{-1} , compressor work, pressure ratio, specific fuel consumption, and engine-cycle efficiency are not included. The reported percentage savings therefore cannot be presented directly as fuel savings or engine-efficiency gains.

5. Validation pathway. The same architecture should next be calibrated against three-dimensional CHT/CFD or an experimental thermal rig. Sensor noise and delay, actuator bandwidth, realistic film/impingement interactions, pressure loss, and coating thermomechanics should then be included [22–24]. The normalized commands can subsequently be linked to actual bleed flow and engine-cycle metrics. Only after these steps would stronger engineering claims about reliability, efficiency, and life be justified.

7. CONCLUSIONS AND RESEARCH PERSPECTIVE

This book chapter combines the Earth-inspired principle of 'layer and manage the thermal gradient' with the skin-inspired principle of 'sense, respond locally, and maintain a protective balance' in a three-level turbine-blade pre-design architecture. The Earth example is used as a way to distinguish the different functions of mantle transport and the conductive upper boundary layer; the skin example provides functional inspiration for state-trend-memory feedback. The equal-resistance one-dimensional FGM demonstration probes gradient redistribution, whereas the three-region ROM probes regional cooling and control logic.

The numerical case study shows clearly that nominal economy and thermal protection under uncertainty are not the same objective. Relative to V2, V4

reduces the normalized coolant integral by 5.01% and peak temperature by 5.773 K, yet it satisfies the three-part success criterion in only 0.5% of the prescribed uncertainty scenarios. V4-R raises the thermal pass fractions to 95% for the peak criterion, 100% for exposure, and 95% jointly, while mean normalized saving falls to 0.44%. These results show why the trade-off between safety margin and economic target should not be compressed into a single 'success percentage.'

V4-R2 illustrates the opposite boundary: in the independent 200-scenario numerical test, mean saving increases to 19.96% while the thermal pass fractions fall to 0%. In the separate 800 s ablation, the structure with thermal memory reduces cumulative exposure by 29.28% and increases normalized coolant use by 4.26% relative to the T-only reference. The strongest outcome of the chapter is the definition of a testable system architecture that links layered protection, regional cooling, thermal memory, and, in future work, health information within the same decision chain. Section 6 identifies the CHT/CFD, thermal-rig, and cycle-integration steps required to transfer this architecture toward realistic engine conditions.

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Centre-of-Mass Coordinate Characterisation of a CAD-Derived Mechanical Assembly for Optical Rotary Encoder Dynamic Modelling

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ABSTRACT

The spatial definition of mass properties is a decisive prerequisite when a computer-aided design assembly is converted into a multibody representation for dynamic analysis. This paper examines the centre-of-mass coordinates of the constituent bodies of a mechanical arrangement developed for the simulation of an optical rotary encoder exposed to controlled mechanical vibration. The analysis is restricted to the geometrical and mass-property layer of the model, with particular attention to the way in which the component centres of mass are expressed in a common global coordinate system before transfer to a Simscape Multibody environment. A single global reference frame and a coincident output coordinate frame are adopted so that component positions, inertia quantities and principal directions remain mutually consistent. The mathematical framework covers the aggregate mass properties, component position vectors, homogeneous coordinate transformations, the assembly centre of mass and the translation of inertia quantities between reference points. The numerical results comprise six functional bodies: electric motor, transmission mechanism, optical rotary encoder, encoder housing, static platform and dynamic platform. Their centres of mass are located exclusively along the longitudinal axis of the assembly, while the two transverse coordinates are zero for every body. The resulting arrangement establishes a compact spatial parameter set for subsequent multibody modelling and gives a transparent basis for checking geometric alignment, reference-frame selection and the physical consistency of imported CAD data.

Keywords – Centre of Mass; Mass Properties; CAD Assembly; Optical Rotary Encoder; Global Coordinate System; Simscape Multibody; Multibody Dynamics; Mechanical Vibration

INTRODUCTION

The continuing evolution of digital technologies and data-oriented engineering has contributed to marked advances in measurement precision, system regulation, and automated operation across contemporary technical and industrial environments. Within this framework, optical encoders serve as precise electromechanical sensing devices employed in automation, control engineering, and metrological systems. According to the type of motion being measured and the intended application, optical encoders are commonly distinguished as linear or rotary devices. Linear encoders determine translational displacement, while rotary encoders (Fig. 1) are intended for the measurement of angular position and rotational motion [1–3].

The operating principle is founded on the controlled alteration of a light beam by a rotating disc incorporating successive regions with differing optical properties, including transparent and opaque or reflective and non-reflective sectors. The disc is firmly attached to the shaft of the observed mechanical element, so that both rotate concurrently and permit continuous monitoring of angular displacement in real time [4,5].



Fig. 1 An optical rotary encoder configuration [6,7]

The operating principle of an optical rotary encoder relies on the interaction of an optical beam with a coded disc mechanically coupled to the measured shaft (Fig. 2). The disc incorporates a prescribed arrangement of transparent and opaque regions, or alternatively reflective and non-reflective sectors, according to the adopted optical configuration. A light source, generally a light-emitting diode (LED), directs a monochromatic beam towards the rotating disc. During rotation, the coded pattern periodically interrupts, transmits, or reflects the incident radiation, thereby producing a modulated optical signal. This variation is registered by photosensitive elements, typically photodiodes or phototransistors, positioned in accordance with the encoder architecture. The detected optical fluctuations are subsequently converted into a sequence of electrical pulses whose frequency, phase relationship, and temporal order provide the information required to establish angular position, direction of rotation, and rotational speed [8,9].

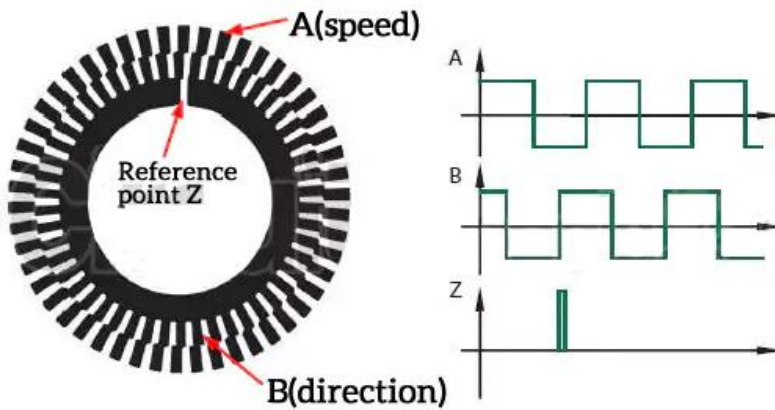


Fig. 2 An optical rotary encoder [10,11]

Rotary encoders installed near drives, transmission elements or machine structures are subjected to oscillatory motion that can alter the relative position of their mechanical and optical parts. The resulting effect is often discussed through signal quality or measurement error, yet the mechanical representation used in a simulation must first reproduce the physical arrangement with sufficient fidelity. Before vibration excitation, solver selection or signal processing can be meaningfully considered, the spatial distribution of mass and the placement of the constituent bodies have to be defined in a common and traceable reference frame.

A CAD model offers a suitable basis for this task because geometry, material assignment, centre-of-mass position and inertia quantities can be evaluated within the same digital description. When the CAD assembly is later transferred to a multibody environment, these quantities become more than descriptive output from the design software: they serve as parameters of rigid-body blocks and influence the dynamic response calculated by the numerical model. The reliability of this transfer depends on coordinate-system consistency. A correct centre-of-mass vector expressed in an unsuitable frame may be numerically precise but physically misplaced after import, while inconsistent frame orientation can introduce unnecessary rotations and obscure the interpretation of inertia quantities.

The investigated mechanical arrangement (Fig. 3) was created as a structured assembly comprising an electric motor, a transmission mechanism, an optical rotary encoder, an encoder housing, a static platform and a dynamic platform. These six bodies form the geometrical basis of the simulation-oriented model. The global coordinate system was selected so that its longitudinal axis follows the common mechanical axis of the rotating elements, whereas the remaining axes span the transverse plane. This arrangement allows the component positions to be interpreted directly in

relation to the mechanical chain and provides a convenient basis for the subsequent transfer from SolidWorks to Simscape Multibody.

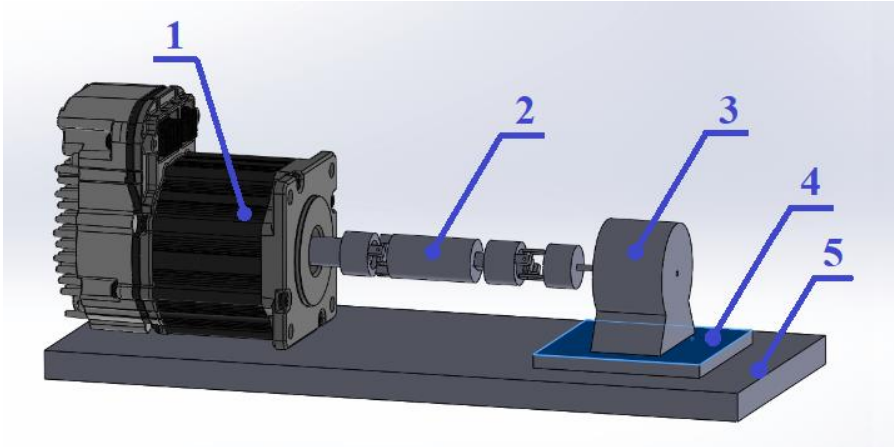


Fig. 3 CAD assembly of the system: 1 – Electric motor; 2 – Transmission mechanism; 3 – Encoder housing;
4 – Dynamic platform; 5 – Static platform [12]

Centre-of-mass coordinates are particularly informative because they condense the spatial distribution of each rigid body into a single position vector. Although they do not replace the inertia tensor or the complete geometry, they provide a direct means of checking whether components are placed on the intended axis, whether transverse offsets are present and whether the chosen reference frame gives a coherent representation of the assembly. In a multibody model, these vectors determine where the mass of each rigid body is effectively concentrated for translational dynamics, while the associated inertia tensor governs resistance to angular acceleration about the body's reference point.

A further reason for isolating the centre-of-mass layer is methodological clarity. In complex simulation studies, geometric modelling, mass-property extraction, kinematic constraints, vibration input, encoder quantisation and error indicators can become intertwined. Treating the mass-centre coordinates as a distinct research object makes it possible to examine one foundational stage of the model in detail. This is valuable when the purpose is not merely to reproduce a software workflow but to explain why the imported model retains a physically interpretable relationship with the original CAD assembly.

The present paper therefore concentrates on the coordinate-based characterisation of the six constituent bodies forming the mechanical assembly, together with the mathematical framework required for their consistent representation within a unified global reference system. The

adopted formulation treats each component as an individual rigid body whose spatial placement is described through a centre-of-mass position vector referenced to the same Cartesian coordinate frame, thereby permitting direct comparison of the component locations without introducing separate local conventions. Particular attention is assigned to the CAD-derived centre-of-mass coordinates, since these quantities provide a concise but physically meaningful description of the effective mass location of each body and constitute an important part of the parameter set required for subsequent multibody modelling. Their evaluation also enables the spatial ordering of the components, the presence or absence of transverse offsets, and the consistency of the selected coordinate architecture to be examined before the assembly is transferred to a dynamic simulation environment. Within this context, the optical rotary encoder represents a central element of the investigated mechanical chain, because its position must remain geometrically consistent with the adjacent motor, transmission, housing and supporting structures if the imported model is to preserve the intended physical arrangement. The relative location of the optical rotary encoder within the complete assembly, together with its spatial relationship to the surrounding mechanical components, is illustrated in Fig. 4.

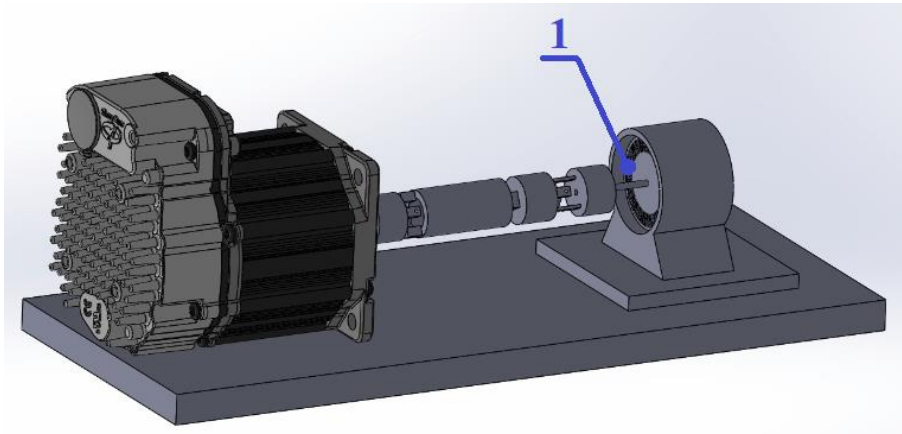


Fig. 4 Position of the optical rotary encoder within the mechanical assembly [9]

The specific objective is to formulate the coordinate and mass-property relations required before numerical results are presented, then to report the six centre-of-mass vectors without embedding equations in the Results section. This separation is deliberate. It allows the Materials and Methods section to contain the full mathematical basis, while the Results section remains a clean record of the extracted CAD quantities. The Discussion then evaluates the geometric meaning of the coordinate pattern, its consequences for model organisation and the limitations that should be recognised when the same approach is extended to more elaborate mechanical assemblies.

The study is relevant beyond the particular encoder model. Any CAD-to-multibody workflow depends on an unambiguous relation between geometry, reference frames and mass properties. The same principles apply to machine-tool subassemblies, actuator systems, sensor fixtures, robotic joints and laboratory test rigs. A concise but rigorous treatment of centre-of-mass coordinates can therefore serve both as a verification stage within the present model and as a transferable procedure for other simulation tasks in which rigid bodies are imported from a CAD environment.

MATERIALS AND METHODS

The methodological procedure was devised to maintain a consistent relationship between the CAD geometry, the component mass properties and the coordinate representation required for subsequent multibody modelling. The mechanical arrangement introduced in Section 1 was constructed in SolidWorks as an assembly of six functionally distinct rigid bodies: the electric motor, transmission mechanism, optical rotary encoder, encoder housing, static platform and dynamic platform. Each component was retained as an individual body so that its spatial position and inertial characteristics could be evaluated independently.

The geometric definitions and material assignments were completed before the Mass Properties functionality was applied. For each body, the extracted quantities comprised mass, volume, surface area, centre-of-mass coordinates, principal moments of inertia and inertia-tensor components. The present analysis concentrates primarily on the centre-of-mass coordinates, since these quantities provide a direct description of the spatial location of each constituent body within the assembled system. The remaining mass properties were retained as supplementary physical parameters required for a coherent rigid-body representation.

The complete assembly has a total mass of 3.60419 kg, obtained from the individual component contributions. The broader mass-property characterisation of the same mechanical assembly, including component masses, volumes, surface areas, inertia tensors and the global centre-of-mass evaluation, has been reported previously [13]; the present study isolates the coordinate-based centre-of-mass description for a more detailed spatial analysis. For a system consisting of n rigid bodies, the aggregate mass and volume may be expressed as:

$$M = \sum_{i=1}^n m_i \quad (1)$$

$$V = \sum_{i=1}^n V_i \quad (2)$$

where m_i and V_i denote the mass and volume of the i -th body, respectively. The aggregate component surface area is similarly defined as:

$$A = \sum_{i=1}^n A_i \quad (3)$$

where A_i represents the surface area of the corresponding rigid body. These relations provide a straightforward check that all intended components have been included in the CAD mass-property evaluation.

A common global Cartesian coordinate system was established for the complete assembly. The coordinate frame employed for reporting the CAD mass properties was defined to coincide geometrically with this global system. Their origins are therefore coincident, while the corresponding axes are collinear and identically oriented. The longitudinal axis follows the principal mechanical axis of the motor, transmission mechanism and optical rotary encoder, whereas the remaining axes span the transverse plane.

The centre of mass of each constituent body is described by a three-component position vector expressed in the global coordinate system. The vector retains both the magnitude and sign of the coordinate values, so that components placed on opposite sides of the origin remain distinguishable.

$$r_{COM,i} = \begin{bmatrix} x_{COM,i} \\ y_{COM,i} \\ z_{COM,i} \end{bmatrix} \quad (4)$$

The assembly centre of mass follows from the mass-weighted combination of the component position vectors. This relation connects the local mass-property output of each body to the overall placement of the assembled system.

$$r_{COM} = \frac{1}{M} \sum_{i=1}^n m_i r_{COM,i} \quad (5)$$

Where a component quantity is initially expressed in its own local frame, a homogeneous transformation provides the general mapping to the global frame. The transformation combines a rotation matrix and a translation vector in a single matrix representation.

$${}^0T_i = \begin{bmatrix} {}^0R_i & {}^0p_i \\ 0^T & 1 \end{bmatrix} \quad (6)$$

The position of the component centre of mass in the global frame may consequently be obtained from the local centre-of-mass vector by the rigid transformation below.

$${}^0r_{COM,i} = {}^0R_i \cdot {}^i r_{COM,i} + {}^0p_i \quad (7)$$

In the configuration employed here, the output and global frames are aligned. For quantities already reported in the coincident output frame, the relevant rotation is the identity matrix. This is the principal reason why the centre-of-mass coordinates can be transferred and interpreted without introducing an additional rotation of the coordinate basis.

$$R = I_3 \quad (8)$$

For subsequent CAD-to-Simscape transfer, each body is represented by a compact parameter set that combines the scalar mass, centre-of-mass position, inertia tensor and geometric description. A convenient formal representation is:

$$P_i = \{m_i, r_{COM,i}, I_i, G_i\} \quad (9)$$

where G_i denotes the CAD geometry associated with the i -th body. In the imported multibody model, these data are assigned to the corresponding rigid-body or Solid representation so that geometry and inertial behaviour remain associated with the same functional component.

The inertia tensor complements the centre-of-mass vector because centre position alone cannot describe the body's resistance to rotation. In a Cartesian frame, the tensor may be written in symmetric matrix form as:

$$I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \quad (10)$$

The diagonal terms are the moments of inertia about the coordinate axes and the off-diagonal terms are the products of inertia. When the coordinate axes coincide with the principal inertia directions, the products of inertia vanish and the tensor reduces to a diagonal matrix. This condition is advantageous in a multibody model because it removes inertial coupling between orthogonal rotational directions at the level of the rigid-body parameterisation.

$$I_{principal} = \text{diag}(I_1, I_2, I_3) \quad (11)$$

When an inertia tensor known at the centre of mass must be expressed about a parallel axis through another reference point, the parallel-axis theorem supplies the required translation. For a displacement vector d from the centre of mass to the new point, the tensor is transformed according to:

$$I_o = I_c + m \left(\|d\|^2 I_3 - dd^T \right) \quad (12)$$

The vector d is defined by the difference between the coordinates of the two reference points. Written component-wise, it contains the displacements along the three global directions.

$$d = \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} \quad (13)$$

The quadratic term entering the parallel-axis relation is therefore:

$$||d||^2 = d_x^2 + d_y^2 + d_z^2 \quad (14)$$

and the dyadic product of the displacement vector is:

$$dd^T = \begin{bmatrix} d_x^2 & d_x d_y & d_x d_z \\ d_y d_x & d_y^2 & d_y d_z \\ d_z d_x & d_z d_y & d_z^2 \end{bmatrix} \quad (15)$$

These relations clarify why a carefully selected reference frame is valuable. If the inertia tensor is reported directly at the centre of mass and the target model uses the same point and axis orientation, the translational correction and rotational change of basis are unnecessary. The imported parameter set then corresponds directly to the CAD result rather than to a subsequently reconstructed equivalent.

The six-body CAD assembly can be formalised as a structured mechanical model containing bodies, coordinate frames, kinematic constraints and parameter sets. In compact notation, the assembly is represented as:

$$A_{CAD} = \{B, J, F, P\} \quad (16)$$

where B is the set of rigid bodies, J the set of kinematic connections or mates, F the set of coordinate frames and P the set of physical parameters. In the present assembly, B contains six elements. The later CAD-to-Simscape conversion maps rigid bodies to Solid blocks, kinematic connections to Joint blocks, coordinate relationships to Rigid Transform blocks and the mass-property data to the inertial fields associated with the rigid-body representation.

A homogeneous transformation also provides a useful formal description of the frame relationship used by the imported multibody model. If two frames are connected without relative rotation, the transformation reduces to a pure translation. For a centre-of-mass frame whose axes remain parallel to the global axes, the matrix takes the simplified form:

$${}^0T_{COM,i} = \begin{bmatrix} I_3 & r_{COM,i} \\ 0^T & 1 \end{bmatrix} \quad (17)$$

Since the centre-of-mass coordinates of all six bodies are expressed in the same global reference frame, their spatial positions can be compared directly without further coordinate transformation. The longitudinal coordinate, $x_{COM,i}$, defines the position of each component relative to the global origin, whereas $y_{COM,i}$ and $z_{COM,i}$ indicate possible transverse offsets from the principal mechanical axis.

The numerical procedure comprised three successive stages. First, the six functional bodies and the common global reference frame were identified within the CAD assembly. Second, the centre-of-mass coordinates calculated by the SolidWorks Mass Properties function were extracted for each body. Third, the obtained values were retained in SI units and arranged according to the corresponding component for subsequent evaluation.

The adopted sign convention follows the orientation of the global Cartesian system. Negative and positive longitudinal coordinates indicate placement on opposite sides of the origin, while a zero transverse coordinate denotes alignment with the corresponding reference plane. All reported centre-of-mass coordinates are expressed in metres.

Before transfer to the multibody model, the extracted coordinate set was examined for geometric consistency. Particular attention was given to the alignment of the component centres with the intended mechanical axis and to their longitudinal ordering within the assembly. This verification provides a direct means of identifying unintended spatial offsets or discrepancies in the CAD-derived mass-property data prior to dynamic modelling.

RESULTS

The centre-of-mass coordinates extracted from the CAD mass-property evaluation are presented in Table 1. Each constituent body is represented by the position vector introduced in (4), whereas all numerical entries are expressed with respect to the common global Cartesian frame established in Section 2. Since the CAD output coordinate system coincides geometrically with the adopted global reference frame, the reported values may be employed directly in accordance with the transformation relation defined by (7) and the identity rotation introduced in (8). The tabulated coordinates therefore constitute the positional component of the rigid-body parameter set formalised in (9).

The coordinate set reveals a strictly longitudinal distribution of all six centres of mass. Variation occurs exclusively along the $x_{COM,i}$ direction, whereas the corresponding $y_{COM,i}$ and $z_{COM,i}$ entries remain equal to zero for every constituent body. Consequently, none of the extracted centres exhibits

a numerical displacement from the two transverse reference planes defined by the global coordinate system.

The electric motor occupies the most negative longitudinal position, with its centre of mass located at -0.05000 m relative to the global origin. The static platform is positioned at -0.03000 m, while the transmission mechanism coincides with the coordinate origin at 0 m. On the positive side of the reference point, the optical rotary encoder is located at 0.05000 m, followed by the encoder housing at 0.09000 m and the dynamic platform at 0.13112 m.

Table 1. Centre-of-mass coordinates of the constituent bodies in the global coordinate system

Body	Constituent element	$\mathbf{r}_{\text{COM},i} = [x_{\text{COM},i}, y_{\text{COM},i}, z_{\text{COM},i}]^T$ [m]
B ₁	Electric motor	$[-0.05000, 0, 0]^T$
B ₂	Transmission mechanism	$[0, 0, 0]^T$
B ₃	Optical rotary encoder	$[0.05000, 0, 0]^T$
B ₄	Encoder housing	$[0.09000, 0, 0]^T$
B ₅	Static platform	$[-0.03000, 0, 0]^T$
B ₆	Dynamic platform	$[0.13112, 0, 0]^T$

Accordingly, the complete set of centres extends over a longitudinal interval of 0.18112 m, measured between the electric motor and the dynamic platform. The negative-coordinate region covers 0.05000 m from the origin to the electric-motor centre, whereas the positive-coordinate region extends to 0.13112 m through the encoder-side portion of the assembly. The coordinate origin occupied by the transmission mechanism consequently separates the two longitudinal regions of the mechanical arrangement.

The numerical spacing between successive centre positions also exhibits a compact distribution along the principal axis. The electric motor and static platform are separated by 0.02000 m. The distance between the static platform and the transmission mechanism is 0.03000 m, while the interval from the transmission mechanism to the optical rotary encoder is 0.05000 m. A further separation of 0.04000 m occurs between the optical rotary encoder and its housing, followed by 0.04112 m between the housing and the dynamic platform.

The optical rotary encoder is therefore positioned within the positive longitudinal portion of the coordinate arrangement rather than at either geometric extreme of the complete system. Its centre is located 0.05000 m from the global origin, 0.04000 m from the encoder housing and 0.08112 m from the dynamic-platform centre. Relative to the two negative-coordinate bodies, the encoder centre is displaced by 0.10000 m from the electric motor and 0.08000 m from the static platform.

Among the six constituent components, the dynamic platform occupies the furthest positive longitudinal position within the adopted reference frame, placing its centre of mass furthest along the positive direction of the adopted reference axis, whereas the electric motor is positioned at the opposite extreme and displays the largest negative displacement. The other four components are distributed between these two limiting positions and therefore occupy intermediate locations within the common coordinate system. Such a spatial arrangement establishes a clearly defined longitudinal distribution of the individual centres of mass and facilitates a systematic interpretation of their relative placement within the mechanical assembly. Accordingly, the coordinate ordering provides an unambiguous sequence of component centre positions extending from the most negative towards the most positive region of the selected reference frame:

$$B1 \rightarrow B5 \rightarrow B2 \rightarrow B3 \rightarrow B4 \rightarrow B6$$

The results also confirm that no additional transverse coordinate correction is required for any component in the reported configuration. In numerical terms, every body satisfies $y_{COM,i}=0$ and $z_{COM,i}=0$, while the complete spatial distinction among the six rigid bodies is retained through the signed longitudinal coordinate $x_{COM,i}$.

When incorporated into the homogeneous representation introduced in Section 2, each coordinate triplet supplies the translational component required by the centre-of-mass frame. Since the corresponding transverse terms are zero, the translation for each body is determined solely by its longitudinal coordinate. The values listed in Table 1 may consequently be introduced directly into the simplified transformation defined by (17), without the need to derive supplementary transverse offsets.

The numerical dataset thus provides six distinct centre-of-mass position vectors associated with the six functional bodies of the CAD assembly. Together, these coordinates establish the spatial parameter layer subsequently required for assigning the corresponding rigid-body positions during CAD-to-Simscape Multibody conversion.

DISCUSSION

The coordinate distribution obtained from the CAD model provides insight into the structural organisation of the mechanical assembly before dynamic excitation is introduced. The most conspicuous feature is the confinement of all centre-of-mass locations to a single longitudinal line. Although the six bodies possess different geometry, function and longitudinal placement, none exhibits a transverse displacement with respect to the selected global frame. This arrangement is consistent with the mechanical concept adopted for the assembly, in which the principal rotational elements are arranged along a common axis and the surrounding structural parts are positioned with respect to the same reference direction.

The zero values of $y_{COM,i}$ and $z_{COM,i}$ are especially relevant from the standpoint of model formulation. These values indicate that the centres of mass of the constituent bodies lie directly on the longitudinal reference axis rather than being laterally or vertically displaced from it. Consequently, the spatial description of the assembly can be reduced, at the centre-of-mass level, to a one-dimensional sequence embedded in a three-dimensional coordinate system. Such a configuration does not eliminate the three-dimensional nature of the mechanical model, but it substantially simplifies the positional representation of the rigid bodies. The relationship between the physical configuration of the mechanical assembly and the corresponding centre-of-mass representation is illustrated in Fig. 5.

This characteristic is advantageous during the transition from CAD geometry to a multibody simulation environment. The methodological formulation in Section 2 assumes a common global coordinate frame together with a geometrically coincident output frame. Under these circumstances, the centre-of-mass coordinates do not require an additional change of basis before they are assigned to the corresponding rigid-body representation. The numerical position obtained in the CAD environment consequently retains the same physical interpretation after import, provided that the frame definition is preserved.

The longitudinal sequence of the six bodies also corresponds to the functional organisation of the assembly. The motor lies on the negative side of the reference origin, while the encoder-related bodies occupy the positive region. The transmission mechanism is located at the global origin and therefore forms a convenient positional reference between the driving and sensing portions of the structure. This arrangement facilitates interpretation of subsequent kinematic and dynamic quantities because component positions can be referenced to a physically meaningful central point rather than to an arbitrary external origin. The resulting longitudinal distribution of the six constituent-body centres, together with their signed positions relative to the global origin, is presented in Fig. 6.

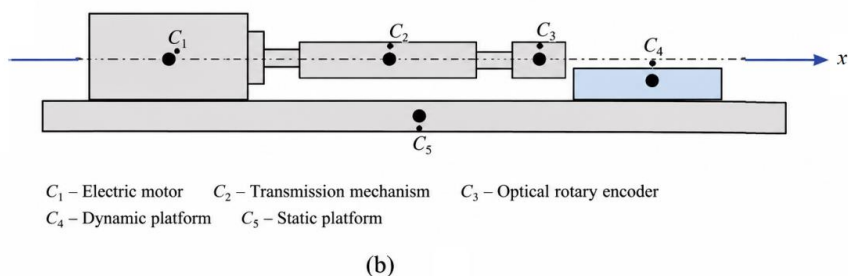
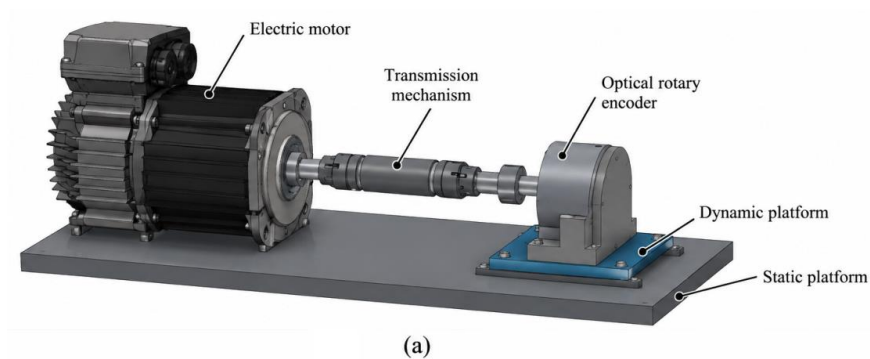


Fig. 5 Physical and centre-of-mass representations of the mechanical assembly: (a) CAD configuration;
 (b) schematic centre-of-mass arrangement

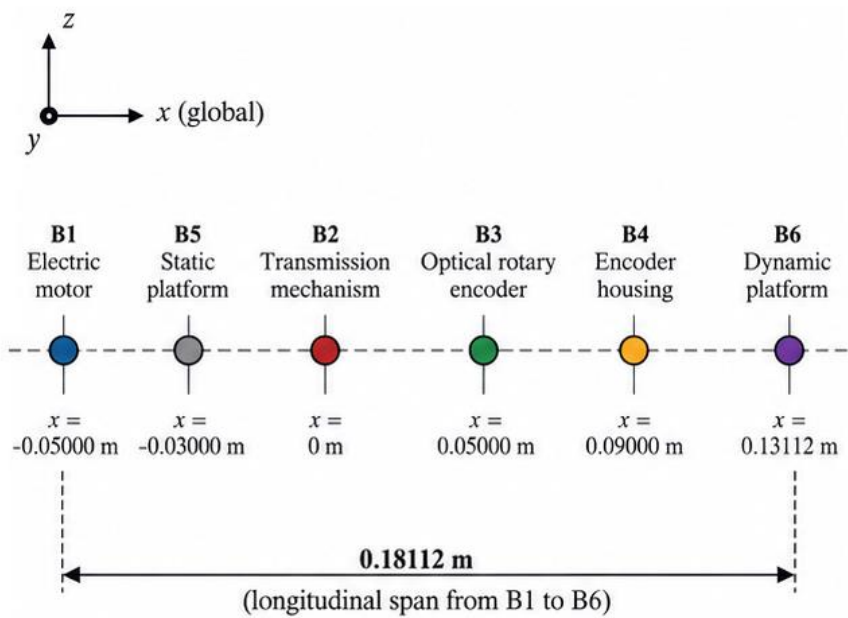


Fig. 6 Longitudinal centre-of-mass distribution of the six constituent bodies

The centre-of-mass location of the optical rotary encoder warrants particular attention because the encoder represents the sensing component around which the broader simulation architecture is organised. Its centre lies at $x=0.05000$ m and remains aligned with the same longitudinal axis as the driving and transmission elements. In geometric terms, this means that the encoder is not represented with a transverse eccentricity in the current CAD configuration. Any subsequent transverse response observed during dynamic simulation could therefore be attributed to the prescribed mechanical motion, joint behaviour, modelling assumptions or imposed excitation rather than to an initial transverse displacement of the encoder centre.

The housing is positioned further along the same axis at 0.09000 m. The relatively small separation of 0.04000 m between the encoder and housing centres reflects their close spatial association within the mechanical subsystem. The dynamic platform follows at 0.13112 m and represents the most distant positive centre-of-mass position. This ordering is useful when tracing the propagation of prescribed motion through the imported multibody architecture because each rigid body retains a distinct signed position relative to the common origin.

The static platform, positioned at -0.03000 m, differs functionally from the rotating components but nevertheless shares the same transverse alignment. Its centre-of-mass location demonstrates that the coordinate convention has been applied consistently not only to the rotational chain but also to the supporting structure. The resulting numerical pattern therefore describes the complete assembly through one coherent spatial reference rather than through separate local conventions for rotating and stationary bodies.

An additional benefit of the obtained arrangement concerns transformation complexity. In a general three-dimensional assembly, each rigid-body centre may require translations along all three axes together with a rotational transformation between local and global coordinate systems. Here, the positional part is considerably more compact because two of the three translational components vanish for every body. Furthermore, the aligned reference frames introduced in the methodology remove the need for an additional rotational change of basis for the reported coordinate quantities. This yields a more transparent CAD-to-multibody mapping and reduces the number of transformation terms that must be inspected during model verification. The resulting CAD-to-multibody parameter-transfer procedure, from the original assembly and mass-property extraction to the common global frame and direct Simscape mapping, is summarised in Fig. 7.

This simplification has practical relevance for error detection during model construction. A non-zero y- or z-coordinate appearing unexpectedly after import would provide an immediate indication of a possible frame-definition inconsistency, unintended CAD displacement or incorrect transformation. The coordinate table can therefore serve not merely as

descriptive output but also as a compact verification dataset. Comparing the original CAD values with the coordinates represented in the multibody model offers a direct check that spatial information has been transferred without an inadvertent offset.

The longitudinal span of 0.18112 m between the electric motor and the dynamic platform further provides a concise measure of the spatial extent covered by the component centres. This quantity does not represent the total geometric length of the physical assembly, since each rigid body possesses finite dimensions beyond its mass centre. Nevertheless, it characterises the interval over which the representative mass locations are distributed and may assist in checking the relative organisation of the imported bodies.

The present findings should also be distinguished from a complete assessment of mass distribution. A centre-of-mass vector provides the effective spatial location of a body's mass but does not convey how that mass is distributed around the point. Two bodies may possess identical centre coordinates while having markedly different inertia tensors and, consequently, different rotational responses. For this reason, the coordinate results should be regarded as one component of the broader rigid-body parameter set defined in Section 2 rather than as a complete description of the dynamic properties.

The inertia tensor introduced in the methodological formulation complements this positional information by quantifying resistance to angular acceleration and, where applicable, inertial coupling between coordinate directions. Hence, the present coordinate analysis verifies the spatial layer of the model, whereas subsequent dynamic calculations depend jointly on mass, centre location, inertia, constraints and externally imposed excitation.

The absence of transverse centre offsets also has implications for the interpretation of future vibration studies conducted with the same mechanical representation. Since the initial centres are collinear, the undeformed configuration does not contain a geometric eccentricity at the centre-of-mass level that could independently introduce lateral asymmetry. Any vibration-induced transverse motion calculated in a subsequent simulation may therefore be examined relative to an initially aligned configuration. This provides a cleaner reference state for distinguishing prescribed excitation from pre-existing geometric displacement.

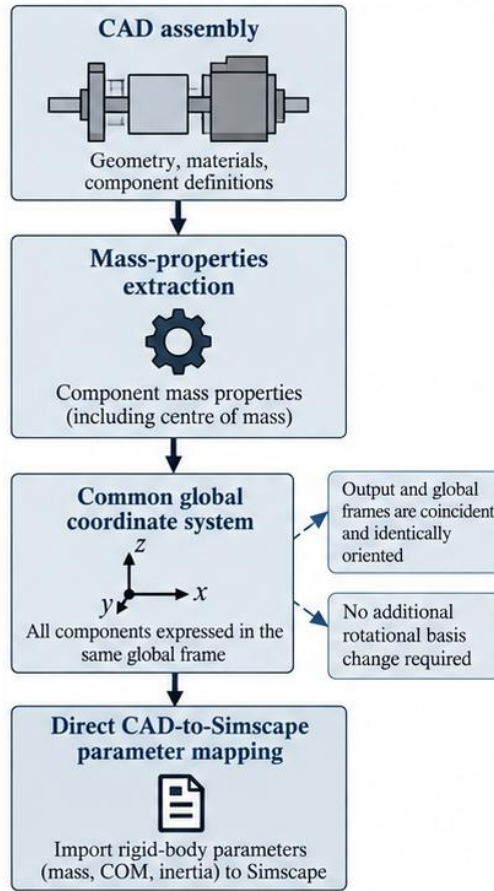


Fig. 7 Schematic workflow of the CAD-to-Simscape mass-property transfer procedure

Nevertheless, the current coordinate pattern should not be interpreted as a universal characteristic of optical-encoder assemblies. The values arise from the specific geometry, material assignments, component arrangement and reference-frame definition adopted in the investigated CAD model. Alternative mounting brackets, asymmetric housings, eccentric couplings or non-coaxial drive layouts would generally produce non-zero transverse centre-of-mass coordinates. In such configurations, the complete three-dimensional form of the transformation relations established in Section 2 would become necessary.

A related limitation concerns the rigid-body assumption associated with the CAD-derived representation. The centre-of-mass coordinates describe the nominal undeformed geometry and remain fixed relative to each rigid body. They do not describe local deformation, structural compliance or redistribution of mass under loading. This assumption is appropriate for a

rigid multibody formulation, although extensions involving flexible bodies or finite-element coupling would require additional information beyond the coordinate dataset considered here.

Material-definition accuracy represents another relevant consideration. CAD mass properties depend on both geometry and assigned material density. An incorrectly specified material would alter the calculated centre of mass for a non-uniform or multi-material component even if its external geometry remained unchanged. Consequently, coordinate verification should be accompanied by confirmation that material assignments correspond to the intended physical components before the mass-property values are transferred to the simulation environment.

Despite these limitations, the six-body coordinate dataset provides a coherent foundation for subsequent modelling. The common coordinate frame, signed longitudinal positions and absence of unintended transverse offsets establish a transparent relationship between the mechanical geometry and its numerical representation. This relationship is particularly important in CAD-to-Simscape workflows, where a geometrically plausible imported model may still contain coordinate inconsistencies that are difficult to recognise through visual inspection alone.

From a modelling perspective, the principal contribution of the present coordinate analysis lies in making the spatial parameterisation explicit before dynamic calculations are undertaken. Instead of treating CAD import as a purely software-driven conversion, the centre-of-mass vectors are examined as physically interpretable quantities whose signs, magnitudes and relative ordering can be independently verified. The procedure therefore strengthens traceability between the original mechanical assembly and the resulting multibody structure.

Taken together, the results indicate that the adopted coordinate architecture produces a compact and internally coherent spatial description of the six constituent bodies. The collinear centre-of-mass arrangement, coincident coordinate frames and direct association between each CAD component and its rigid-body parameter set reduce unnecessary transformation complexity while preserving the physical identity of the mechanical elements. This constitutes an appropriate basis for the subsequent incorporation of joints, prescribed rotational motion and mechanical vibration into the broader simulation framework.

CONCLUSION

This study has established a structured coordinate-based description of the mass-property arrangement of a CAD-derived mechanical assembly intended for subsequent multibody modelling of an optical rotary encoder system. By concentrating specifically on the centre-of-mass coordinates of the constituent rigid bodies, the work isolates a fundamental stage of model

preparation that is often treated as a routine output of CAD software, despite its direct influence on the physical integrity of the resulting simulation architecture.

The adopted procedure enabled the six functional components of the assembly to be expressed within a single global Cartesian reference frame, thereby preserving a coherent spatial relation between the electric motor, transmission mechanism, optical rotary encoder, encoder housing, static platform and dynamic platform. The numerical coordinate set demonstrates that the centres of mass are distributed exclusively along the longitudinal axis of the mechanical arrangement, while the two transverse coordinates remain equal to zero for every body. Such a configuration confirms the intended axial organisation of the assembly at the level of mass-centre placement and provides a concise geometric basis for subsequent rigid-body parameterisation.

A principal merit of the proposed representation lies in the direct correspondence between the CAD-derived quantities and the parameters required within a multibody simulation environment. Since the adopted output and global coordinate systems are coincident and identically oriented, the extracted centre-of-mass vectors may be transferred without additional rotational transformation. This reduces the number of intermediate coordinate operations, simplifies the verification procedure and diminishes the likelihood of introducing artificial offsets during CAD-to-Simscape conversion.

The centre-of-mass coordinates also offer an effective means of checking the internal consistency of the mechanical model before dynamic excitation is introduced. Their signed longitudinal positions provide an immediate indication of the spatial ordering of the rigid bodies relative to the global origin, whereas the absence of transverse displacement permits straightforward identification of any unintended lateral or vertical offset that might arise during model construction or import. In this sense, the coordinate dataset serves not merely as descriptive numerical output but as a verification layer connecting geometric design with simulation-ready physical parameters.

The analysis further demonstrates the value of separating the positional description of the rigid bodies from the remaining dynamic quantities. The centre-of-mass vector defines where the mass of each constituent element is effectively located, but does not independently determine its rotational behaviour. A complete rigid-body description additionally requires mass, inertia tensor, geometry and kinematic relations. Nevertheless, the coordinate layer forms an indispensable part of this broader parameter set, because any inconsistency in the reference-frame definition would propagate into the subsequent dynamic model regardless of the numerical accuracy of the remaining inertial quantities.

The resulting framework therefore provides a transparent foundation for the later integration of joint constraints, rotational input, vibration excitation and encoder-related dynamic phenomena. By establishing the undeformed spatial configuration before these effects are introduced, the model acquires a clearly defined reference state from which subsequent translational and rotational responses can be interpreted. This is particularly important for vibration-oriented studies, since dynamic deviations can then be distinguished from errors associated with initial geometric misalignment or inconsistent coordinate conventions.

Although the investigated assembly exhibits a comparatively simple axial centre-of-mass arrangement, the methodological approach is not restricted to this configuration. The same procedure may be extended to systems containing eccentric bodies, asymmetrical housings, non-coaxial transmission elements or more elaborate three-dimensional support structures. In such cases, non-zero transverse coordinates and additional rotational transformations would have to be retained, but the underlying workflow would remain applicable: define a common reference frame, extract the relevant mass properties, establish the centre-of-mass vectors and verify their consistency prior to multibody import.

Certain limitations should nevertheless be recognised. The current analysis is based on the nominal CAD geometry and the associated rigid-body mass properties. It does not account for elastic deformation, manufacturing tolerances, assembly clearances, material inhomogeneity or structural compliance. Furthermore, the centre-of-mass coordinates remain fixed relative to each rigid body and therefore do not describe local deformation under dynamic loading. These aspects would require complementary modelling strategies, such as flexible multibody formulations, finite-element coupling or experimentally identified geometric corrections.

Future development may therefore proceed in several directions. One extension could involve direct numerical comparison between the CAD-derived coordinate set and the corresponding parameters after import into Simscape Multibody, thereby providing an explicit transfer-validation procedure. A further step could incorporate experimentally measured component positions to quantify any difference between the nominal CAD configuration and the assembled physical system. The methodology could also be expanded to include automated consistency checks for centre-of-mass location, inertia tensor orientation and reference-frame definition as part of a broader CAD-to-simulation verification workflow.

From an engineering standpoint, the principal outcome is the establishment of a compact and physically interpretable coordinate representation that preserves traceability between the mechanical design and the computational model. The six centre-of-mass vectors form a coherent spatial dataset, while the coincident coordinate architecture enables those values to be introduced into the multibody environment without unnecessary reconstruction. This

enhances transparency in model preparation and provides a firmer basis for assessing the validity of subsequent dynamic simulations.

Overall, the work demonstrates that centre-of-mass characterisation should be regarded as a deliberate modelling stage rather than as an auxiliary CAD output. When the coordinate system, body positions and mass-property definitions are treated consistently, the resulting multibody representation becomes easier to verify, interpret and extend. The approach developed here consequently provides a suitable foundation for simulation studies in which optical rotary encoders and associated mechanical components are subjected to prescribed motion and vibration, while also offering a transferable procedure for other CAD-based mechanical systems in which accurate spatial parameterisation is essential.

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A Mass-Properties-Based Approach to the Dynamic Simulation of Optical Rotary Encoders Under Mechanical Vibrations

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ABSTRACT

This paper presents a mass-properties-based approach for the dynamic simulation of optical rotary encoder assemblies subjected to mechanical vibrations. The proposed formulation treats the mass and inertial characteristics of the mechanical structure as active parameters of the numerical model rather than as supplementary CAD information. The analysed assembly comprises an electric motor, transmission mechanism, optical rotary encoder, encoder housing, static platform and dynamic platform. For each rigid body, the relevant properties include mass, volume, surface area, centre-of-mass position, principal inertia axes and inertia tensor components. The complete assembly has a mass of 3.60419 kg, with the individual component masses reproducing the same global value. The centres of mass are distributed exclusively along the longitudinal axis, while the transverse coordinates remain zero. The global centre of mass coincides with the selected coordinate origin, providing a balanced reference configuration for dynamic analysis. The principal inertia axes are aligned with the Cartesian coordinate system, and all products of inertia are zero, resulting in diagonal inertia tensors without inertial cross-coupling. The complete assembly exhibits a longitudinal principal moment of inertia of $0.00357 \text{ kg}\cdot\text{m}^2$ and equal transverse values of $0.02338 \text{ kg}\cdot\text{m}^2$. The results further demonstrate that component position can considerably alter the inertia referred to the global frame, particularly for bodies located farther from the assembly centre. The developed approach provides a consistent basis for transferring CAD-derived physical properties into multibody vibration simulations of optical rotary encoder systems.

Keywords – Optical Rotary Encoder; Mass Properties; Mechanical Vibrations; Dynamic Simulation; Multibody Dynamics; Inertia Tensor; Centre of Mass; CAD Model

INTRODUCTION

An optical encoder is a precision electromechanical sensing device widely applied in automation, motion control and metrological systems. According to the type of displacement being observed, the measurement principle and the form of the generated output, optical encoders are commonly divided into two principal groups: rotary and linear encoders [1–4].

Optical rotary encoders, exemplified by the configuration shown in Fig. 1, are used in positioning and motion-control arrangements where angular displacement and rotational speed must be measured with high repeatability. Their performance is often considered from electrical, optical or signal-processing viewpoints, whereas the mechanical characteristics of the surrounding structure receive less attention. Under mechanical vibration,

however, the encoder cannot be treated as an isolated sensing element, since its response depends on the structural path through which excitation is transmitted. This path may include the motor, transmission mechanism, encoder body, housing and supporting platforms.

The primary function of an optical rotary encoder is to convert shaft rotation into electrical signals suitable for subsequent processing by control or measurement equipment. From these signals, angular position, rotational speed and direction of motion can be determined. Their resolution, repeatability and operational stability make optical encoders appropriate for applications requiring accurate rotational measurement [5].



Fig. 1 An optical rotary encoder [6]

The operating principle is based on the interaction between a light beam and a coded disc mechanically coupled to the rotating shaft. The disc contains transparent and opaque, or reflective and non-reflective, regions arranged according to a prescribed pattern (Fig. 2). During shaft rotation, light emitted by an LED is modulated by these regions and detected by photodetectors. The resulting optical variations are converted into electrical pulses and interpreted to determine rotational motion parameters [7].

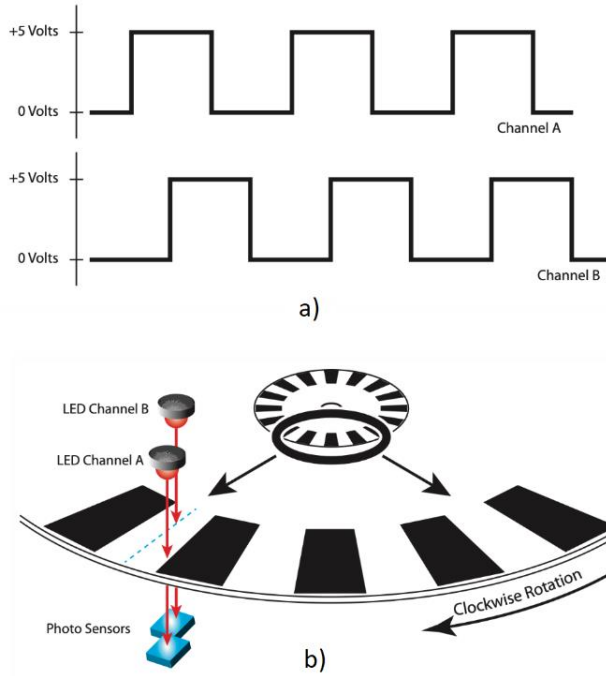


Fig. 2 Angular position representation based on encoder resolution [8]

A mechanically representative simulation of such an arrangement requires more than the reproduction of its external geometry. The distribution of mass, the location of the centre of mass and the rotational inertia of each component govern its response to translational and rotational excitation. These characteristics affect accelerations, reaction forces and moments at the mechanical interfaces and consequently influence the relative motion transmitted to the encoder. A model with geometrically correct components but inconsistent inertial properties may therefore yield a dynamic response that does not adequately represent the physical assembly.

The modelling procedure adopted in this study describes the mechanical arrangement as a system of rigid bodies whose inertial characteristics are obtained from their three-dimensional CAD definitions. Each component is characterised through its mass, volume, surface area, centre-of-mass coordinates, principal axes and inertia tensor. In this manner, the geometric description of the assembly is translated into the parameter set required for dynamic simulation.

The investigated arrangement consists of six functional components: an electric motor, a transmission mechanism, an optical rotary encoder, an encoder housing, a static platform and a dynamic platform (Fig. 3). Their masses, spatial locations and inertia characteristics differ substantially. The electric motor represents the largest individual

contribution to the overall mass, whereas the encoder (Fig. 4) possesses considerably lower mass and moments of inertia. The supporting elements occupy different positions along the longitudinal direction and therefore influence the spatial distribution of mass throughout the complete assembly.

An important characteristic of the configuration is the alignment of the individual centres of mass along a common longitudinal axis. The transverse centre-of-mass coordinates are zero for all components, while the principal inertia axes coincide with the Cartesian coordinate system. In addition, the products of inertia are zero. This arrangement allows the simulation model to preserve the relevant inertial properties of the CAD assembly without introducing artificial coupling between orthogonal rotational directions.

Such alignment is particularly useful when mechanical vibration is introduced through the moving support. The imposed translational disturbance propagates through the multibody structure according to the mass, spatial position and moments of inertia of the individual components. Consequently, the response observed at the encoder arises from the mechanical configuration of the assembly rather than from an independently prescribed disturbance acting directly on the sensing element.

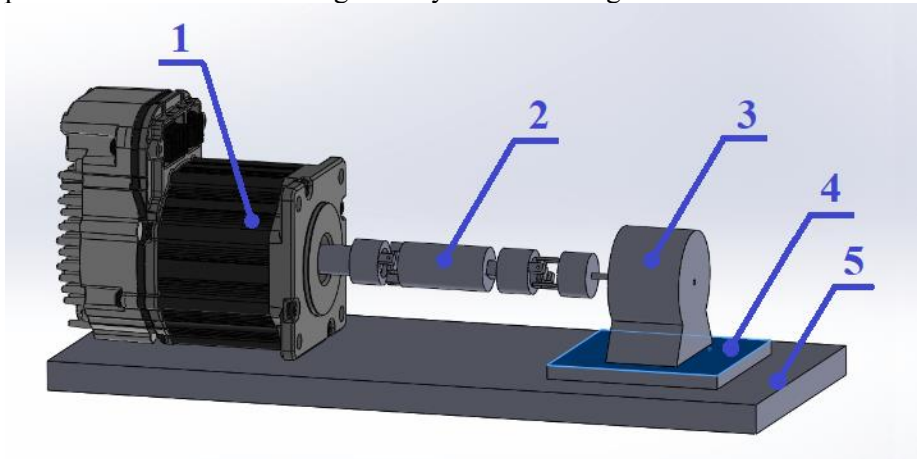


Fig. 3 CAD assembly of the system: 1 – Electric motor; 2 – Transmission mechanism; 3 – Encoder housing; 4 – Dynamic platform; 5 – Static platform [9]

The aim of this work is to develop and assess a dynamic modelling procedure for optical rotary encoders in which the mass and inertial characteristics of the complete mechanical arrangement form the principal physical parameter set. The procedure establishes a direct link between a CAD-defined assembly and a multibody representation intended for vibration analysis. Particular attention is devoted to the consistency of the mass distribution, the coordinates of the component centres of mass, the principal moments of inertia and the corresponding inertia tensors. Within

the analysed mechanical arrangement, the location of the optical rotary encoder relative to the structural components is shown in Fig. 4.

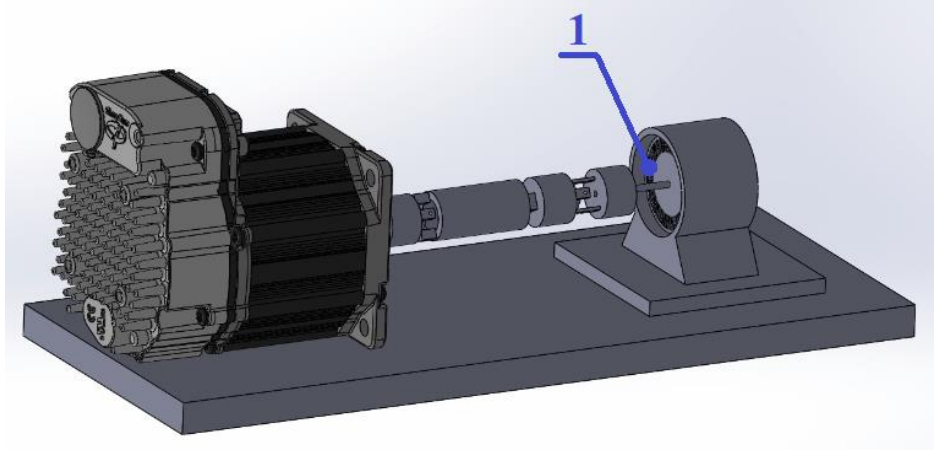


Fig. 4 Position of the optical rotary encoder within the mechanical assembly [9]

Within this framework, mass properties are not treated merely as descriptive output obtained from the CAD environment, but as governing inputs of the dynamic model. Their numerical evaluation provides a concise basis for examining the symmetry, balance and inertial consistency of the mechanical representation before its response under vibration is analysed.

MATERIALS AND METHODS

The mechanical system was constructed as a rigid multibody assembly comprising six functional bodies. The arrangement consisted of the electric motor, transmission mechanism, optical rotary encoder, encoder housing, static platform and dynamic platform. Each component was assigned a material consistent with its intended mechanical role and geometrical construction.

The electric motor was represented using grey cast iron, the transmission mechanism using alloy structural steel, the optical rotary encoder using a zinc alloy, the encoder housing using structural steel, the static platform using a zinc alloy and the dynamic platform using a lead alloy. The material definitions, together with the solid geometry of each body, provided the basis for calculation of the corresponding physical properties.

The fundamental parameter set assigned to an individual rigid body may be expressed as:

$$P_i = \{m_i, r_{COM}, I_i, G_i\} \quad (1)$$

where m_i denotes the mass of body i , r_{COM} is its centre-of-mass position vector, I_i is its inertia tensor and G_i represents its geometric description.

The centre-of-mass position was written as:

$$r_{COM} = \begin{bmatrix} x_{COM} \\ y_{COM} \\ z_{COM} \end{bmatrix} \quad (2)$$

For the complete mechanical assembly, the global centre of mass follows from:

$$r_{COM} = \frac{1}{M} \sum_{i=1}^n m_i r_{COM,i} \quad (3)$$

with:

$$M = \sum_{i=1}^n m_i \quad (4)$$

The aggregate mass obtained from the six component bodies was 3.60419 kg. This value was also obtained directly for the complete assembly, allowing the component-level description to be checked against the global CAD result.

The inertia tensor of each rigid body was represented in Cartesian coordinates as:

$$I_i = \begin{bmatrix} I_{xx,i} & I_{xy,i} & I_{xz,i} \\ I_{yx,i} & I_{yy,i} & I_{yz,i} \\ I_{zx,i} & I_{zy,i} & I_{zz,i} \end{bmatrix} \quad (5)$$

For the analysed geometry, all non-diagonal terms were zero. The tensor therefore reduced to:

$$I_{COM} = \begin{bmatrix} I_{xx,i} & 0 & 0 \\ 0 & I_{yy,i} & 0 \\ 0 & 0 & I_{zz,i} \end{bmatrix} \quad (6)$$

The absence of products of inertia indicates coincidence between the selected coordinate directions and the principal inertia directions. The principal-axis vectors were consequently identical for every analysed body:

$$\begin{aligned} e_x &= (1, 0, 0), & e_y &= (0, 1, 0), \\ e_z &= (0, 0, 1) \end{aligned} \quad (7)$$

This coordinate arrangement is particularly suitable for a multibody simulation because rotation about one principal axis does not introduce an inertial coupling term associated with the remaining orthogonal axes.

When an inertia tensor is referred from a component centre of mass to the assembly reference frame, the positional contribution was evaluated by the parallel-axis theorem. For a component whose centre of mass is displaced only along the x-axis:

$$I_{yy} = L_{yy} + md_x^2 \quad (8)$$

and

$$I_{zz} = L_{zz} + md_x^2 \quad (9)$$

Because displacement along the x-axis does not change the moment about that same axis, the longitudinal component remains:

$$I_{xx} = L_{xx} \quad (10)$$

The dynamic model may generally be represented by the constrained multibody relation:

$$M(q)\ddot{q} + h(q, \dot{q}) + \Phi_q^T \lambda = Q \quad (11)$$

where M is the mass matrix assembled from the individual rigid-body properties, q is the vector of generalised coordinates, h contains velocity-dependent and other dynamic terms, $\Phi_q^T \lambda$ represents forces associated with kinematic constraints and Q denotes the applied generalised forces.

Within this formulation, the values extracted from the CAD mass properties are not merely auxiliary parameters. They directly contribute to the coefficients of the governing dynamic system. Alteration of component mass, centre-of-mass location or inertia therefore modifies the simulated response even when the external geometry and prescribed excitation remain unchanged.

Mechanical vibration can be introduced through displacement of the dynamic platform. For harmonic excitation, the imposed motion may be written as:

$$y_v(t) = A_v \sin(2\pi f_v t) \quad (12)$$

where A_v is vibration amplitude and f_v is vibration frequency. Corresponding velocity and acceleration are:

$$\dot{y}_v(t) = 2\pi f_v A_v \cos(2\pi f_v t) \quad (13)$$

and:

$$\ddot{y}_v(t) = -(2\pi f_v)^2 A_v \sin(2\pi f_v t) \quad (14)$$

The acceleration term becomes increasingly significant as vibration frequency rises and therefore interacts directly with the mass and inertia of the participating bodies. Consequently, even relatively small imposed displacements may create appreciable inertial loading at elevated frequencies.

The geometrical assembly can be transferred to a multibody environment by preserving the rigid-body geometry, coordinate frames, mechanical constraints and mass-property data. In such a representation, the individual components become rigid bodies, mechanical connections become joints or rigid transformations, and the calculated masses and inertia tensors are assigned to their respective dynamic bodies.

This procedure retains the physical distinction between the excitation source and the encoder response. The dynamic platform receives the prescribed vibration, whereas the motion of the remaining bodies follows from the constrained mechanical system. The encoder is thereby subjected to the resulting mechanical response of the assembly rather than to an independently imposed mathematical error.

RESULTS

The numerical mass properties of the complete assembly and the six constituent components, corresponding to the rigid-body parameter set defined in Eq. (1), are presented in Table 1. The values reveal substantial differences in mass, spatial position and rotational inertia between the elements of the mechanical system.

The mass distribution is strongly non-uniform. The electric motor, with a mass of 1.65 kg, represents approximately 45.8% of the complete assembly. The static platform contributes approximately 15.7%, the dynamic platform 13.0%, the transmission mechanism 11.7%, the encoder housing 8.9% and the optical rotary encoder approximately 5.0%.

This distribution is dynamically important. Although the optical encoder is the component of primary metrological interest, it does not dominate the inertia of the mechanical arrangement. The dynamic state experienced at the encoder location results from the interaction of considerably heavier surrounding elements. A reduced model based exclusively on encoder mass would therefore omit a substantial part of the mechanical environment through which vibration is transmitted.

As a consistency check, application of the total-mass relation in Eq. (4) gives 3.60419 kg for the six component bodies, exactly matching the complete assembly. The corresponding component volumes sum to 0.000470 m³ and the surface areas to 0.14164 m², again reproducing the

assembly values. This agreement confirms that the rigid-body decomposition preserves the physical content of the CAD structure.

The centre-of-mass vectors defined by Eq. (2) are equally informative. All yCOM and zCOM coordinates are zero, while the component centres vary exclusively along the x-direction. The electric motor is located at -0.05 m relative to the reference origin, the transmission mechanism at 0 m, the encoder at 0.05 m, the encoder housing at 0.09 m, the static platform at -0.03 m and the dynamic platform at 0.13112 m.

Table 1. Geometrical and mass properties of the mechanical assembly and its constituent components

Parameter	Mechanical assembly	Electric motor	Transmission mechanism	Optical rotary encoder	Encoder housing	Static platform	Dynamic platform
Material	—	Grey cast iron	Alloy structural steel	Zinc alloy	Structural steel	Zinc alloy	Lead alloy
m [kg]	3.60419	1.65000	0.42000	0.18000	0.32000	0.56419	0.47000
V [m ³]	0.000470	0.000215	0.000053	0.000031	0.000041	0.000088	0.000042
A [m ²]	0.14164	0.05500	0.02800	0.00850	0.01100	0.02500	0.01414
x _{COM} [m]	0	−0.05000	0	0.05000	0.09000	−0.03000	0.13112
y _{COM} [m]	0	0	0	0	0	0	0
z _{COM} [m]	0	0	0	0	0	0	0
d _x [m]	0	−0.05000	0	0.05000	0.09000	−0.03000	0.13112
d _y [m]	0	0	0	0	0	0	0
d _z [m]	0	0	0	0	0	0	0

Using the mass-weighted relations in Eqs. (3) and (4), the longitudinal centre-of-mass calculation gives a numerator of approximately 7×10^{-7} kg·m. Within the numerical precision of the model, the complete assembly centre of mass is therefore located at (0, 0, 0).

This result does not arise from identical component positions, but from their balanced mass-weighted distribution. The relatively heavy electric motor at $x = -0.05$ m is counterbalanced by the encoder, housing and dynamic platform on the positive side of the longitudinal axis.

Such balance is particularly desirable when the assembly is exposed to mechanical vibration. If the global centre of mass were considerably displaced from the selected mechanical reference, translational excitation could introduce additional rotational tendencies through the corresponding moment arms. Positioning the global centre of mass at the coordinate origin provides a mechanically coherent reference for dynamic calculations and reduces the possibility of artificial coupling introduced solely by the coordinate definition.

The principal-axis vectors obtained for the assembly and for every constituent component are identical to the Cartesian directions defined in Eq. (7), as summarised in Table 2.

Table 2. Principal inertia axes

Axis	Mechanical assembly and all component bodies
I_x	(1, 0, 0)
I_y	(0, 1, 0)
I_z	(0, 0, 1)

The result shows complete alignment between the principal inertia directions and the Cartesian coordinate system, as represented by Eq. (7). No rotational transformation of the inertia tensor is therefore required when the properties are assigned to the rigid bodies in the multibody model.

The principal moments of inertia are listed in Table 3:

Table 3. Principal moments of inertia

Parameter	Mechanical assembly	Electric motor	Transmission mechanism	Optical rotary encoder	Encoder housing	Static platform	Dynamic platform
P_x	0.00357	0.00110	0.00030	0.00008	0.00020	0.00120	0.00069
P_y	0.02338	0.00166	0.00234	0.00020	0.00076	0.00195	0.00080
P_z	0.02338	0.00166	0.00234	0.00020	0.00076	0.00195	0.00080

For the complete assembly, the longitudinal principal moment is $0.00357 \text{ kg}\cdot\text{m}^2$, whereas the two transverse principal moments are equal at $0.02338 \text{ kg}\cdot\text{m}^2$. The transverse-to-longitudinal inertia ratio is therefore approximately 6.55.

Accordingly, the assembly has considerably lower rotational inertia about its longitudinal axis than about either transverse direction. This distinction is relevant when rotational perturbations arise from vibration or reaction forces within the kinematic chain; the system is mechanically more resistant to angular acceleration about the transverse axes.

The equality $P_y = P_z$ indicates identical rotational inertia about the two transverse principal axes. The same relationship is present for the transmission mechanism and the optical rotary encoder.

The optical rotary encoder has a particularly small longitudinal principal moment of $0.00008 \text{ kg}\cdot\text{m}^2$, while both transverse moments are $0.00020 \text{ kg}\cdot\text{m}^2$.

The small absolute values reflect the limited mass and compact dimensions of the encoder. Consequently, comparatively modest mechanical moments can produce measurable angular acceleration of the encoder body if such motion is not sufficiently constrained by its mounting arrangement.

The electric motor has a longitudinal principal moment of $0.00110 \text{ kg}\cdot\text{m}^2$ and equal transverse moments of $0.00166 \text{ kg}\cdot\text{m}^2$. Its substantial mass makes it a major contributor to the overall dynamic response, although the global inertia cannot be obtained by simply summing local principal moments because component offsets add positional contributions.

The dynamic platform illustrates this effect particularly clearly. Its local principal moment is $0.00069 \text{ kg}\cdot\text{m}^2$ about the longitudinal axis and $0.00080 \text{ kg}\cdot\text{m}^2$ about each transverse axis, while its centre lies at $x = 0.13112 \text{ m}$, the largest longitudinal offset among the components. Its contribution to global transverse inertia therefore increases markedly when its position is taken into account.

This positional influence becomes evident by comparing the local inertia tensors, represented by the Cartesian tensor form in Eq. (5) and listed in Table 4, with the corresponding tensors referred to the assembly reference frame in Table 5.

All off-diagonal components reported in Table 4 are equal to zero, indicating the absence of inertial coupling between the three mutually orthogonal coordinate directions. Accordingly, the local inertia tensor of each rigid body assumes the diagonal form expressed by Eq. (6). This configuration confirms that the selected Cartesian axes coincide with the principal inertia directions of the individual components, which simplifies their subsequent implementation within the multibody model. It also permits the rotational behaviour of each body to be interpreted independently with respect to its principal axes, without introducing additional coordinate transformations or cross-axis inertia terms.

Table 4. Local inertia tensor components L

Parameter [$kg \cdot m^2$]	Mechanical assembly	Electric motor	Transmission mechanism	Optical rotary encoder	Encoder housing	Static platform	Dynamic platform
L_{xx}	0.00357	0.00110	0.00030	0.00008	0.00020	0.00120	0.00069
L_{xy}	0	0	0	0	0	0	0
L_{xz}	0	0	0	0	0	0	0
L_{yx}	0	0	0	0	0	0	0
L_{yy}	0.02338	0.00166	0.00234	0.00020	0.00076	0.00195	0.00080
L_{yz}	0	0	0	0	0	0	0
L_{zx}	0	0	0	0	0	0	0
L_{zy}	0	0	0	0	0	0	0
L_{zz}	0.02338	0.00166	0.00234	0.00020	0.00076	0.00195	0.00080

While Table 4 characterises the inertia properties evaluated about the individual component centres of mass, a further representation is required to examine the influence of component position within the complete mechanical assembly. For this purpose, the inertia tensors are referred to the common assembly reference frame by accounting for the corresponding centre-of-mass offsets. The resulting values, which combine the local inertia of each body with the positional contribution arising from its location in the assembly, are presented in Table 5.

Table 5. Inertia tensor components referred to the assembly reference frame

Parameter [$kg \cdot m^2$]	Mechanical assembly	Electric motor	Transmission mechanism	Optical rotary encoder	Encoder housing	Static platform	Dynamic platform
I_{xx}	0.00357	0.00110	0.00030	0.00008	0.00020	0.00120	0.00069
I_{xy}	0	0	0	0	0	0	0
I_{xz}	0	0	0	0	0	0	0
I_{yx}	0	0	0	0	0	0	0
I_{yy}	0.02338	0.00579	0.00234	0.00065	0.00335	0.00246	0.00879
I_{yz}	0	0	0	0	0	0	0
I_{zx}	0	0	0	0	0	0	0
I_{zy}	0	0	0	0	0	0	0
I_{zz}	0.02338	0.00579	0.00234	0.00065	0.00335	0.00246	0.00879

For the complete assembly, the reference-frame inertia tensor is diagonal in accordance with Eq. (6), with $I_{xx} = 0.00357 \text{ kg} \cdot \text{m}^2$ and $I_{yy} = I_{zz} = 0.02338 \text{ kg} \cdot \text{m}^2$.

Comparison of the local and reference-frame tensors further clarifies the influence of spatial positioning. The transverse reference-frame values are obtained from the parallel-axis relations in Eqs. (8) and (9), whereas the longitudinal value remains unchanged according to Eq. (10). For the transmission mechanism, whose centre of mass lies at the coordinate origin, the local and reference-frame tensors are identical; the transverse value remains $0.00234 \text{ kg} \cdot \text{m}^2$ in both descriptions.

The encoder, positioned 0.05 m from the origin, has a local transverse inertia of $0.00020 \text{ kg} \cdot \text{m}^2$, whereas the corresponding reference-frame value increases to $0.00065 \text{ kg} \cdot \text{m}^2$. The increase follows from its finite mass and longitudinal offset.

For the encoder housing, the local transverse inertia is $0.00076 \text{ kg} \cdot \text{m}^2$, while the reference-frame value is $0.00335 \text{ kg} \cdot \text{m}^2$.

Its centre of mass lies 0.09 m from the reference origin, so the positional contribution to transverse rotational inertia is larger than the local inertia of the component itself.

Using Eq. (8), the dynamic platform exhibits a reference-frame transverse inertia of $0.00879 \text{ kg} \cdot \text{m}^2$, which is almost eleven times its local transverse inertia and reflects the pronounced influence of its longitudinal position within the assembly.

The electric motor provides a similar illustration. Its local transverse inertia is $0.00166 \text{ kg} \cdot \text{m}^2$, while the tensor referred to the assembly system gives $0.00579 \text{ kg} \cdot \text{m}^2$. The difference is attributable to its mass of 1.65 kg and its -0.05 m centre-of-mass offset. The electric motor therefore exerts an

appreciable influence on transverse dynamic behaviour despite being located relatively close to the global centre.

These results distinguish two mechanical contributions that would otherwise be difficult to separate. The local inertia describes resistance to angular acceleration about axes passing through the component centre of mass, whereas the reference-frame inertia incorporates both local shape-dependent inertia and the spatial placement of that body within the assembly.

Such distinction is especially relevant for vibration simulation. Within the constrained multibody formulation of Eq. (11), the harmonic platform motion and its kinematic derivatives are prescribed by Eqs. (12)–(14). The excitation therefore does not act on six independent components located at their individual centres of mass, but on a spatially connected structure. Dynamic forces are transmitted through joints and rigid connections, and the resulting accelerations depend simultaneously on local body inertia and global configuration.

The encoder itself contributes only 0.18 kg to the total mass. Nevertheless, its centre of mass lies at $x = 0.05$ m and its measurement function is associated with the rotational motion transmitted through the mechanical chain. Its comparatively low inertia means that the encoder body can respond readily to perturbing moments, while its location prevents the response from being considered independently of the neighbouring transmission mechanism and housing.

The zero products of inertia in both tensor descriptions are another significant characteristic. Because all cross-axis components are zero, the inertia tensor retains the uncoupled diagonal form given in Eq. (6) with respect to the principal directions.

This property provides a comparatively transparent basis for analysing vibration-induced encoder disturbances. Any cross-axis motion arising during simulation can principally be associated with the kinematic constraints, external excitation or multibody geometry rather than with an artificial rotation of the inertia coordinate frame.

DISCUSSION

The obtained results indicate that the mechanical behaviour of the analysed encoder assembly cannot be represented adequately through component mass alone. The numerical data show that the dynamic identity of each rigid body is governed by a combined set of properties comprising mass, centre-of-mass location and rotational inertia. These quantities operate jointly within the multibody formulation and determine how the individual elements respond to imposed motion, transmit inertial loads and interact through their mechanical connections. The mass-properties-based formulation therefore offers a more physically representative description of the encoder

environment than a reduced model in which the sensing element is considered independently from the surrounding structure.

A particularly relevant outcome concerns the non-uniform distribution of mass within the assembly. The electric motor constitutes the largest individual contribution to the total mass, whereas the optical rotary encoder represents only a small fraction of the complete system. This disparity has direct mechanical implications. Although the encoder is the principal component from the measurement perspective, its motion is influenced by considerably heavier neighbouring bodies whose inertial response governs the transmission of vibration through the assembly. Consequently, the disturbance experienced at the encoder cannot be interpreted solely from the encoder mass or its local inertia. The surrounding mechanical structure forms an integral part of the excitation path and must therefore remain included in the dynamic representation.

The calculated centre-of-mass coordinates provide a further indication of the internal balance of the system. All component centres lie along the longitudinal axis, while the transverse coordinates remain zero. Despite the different longitudinal positions of the individual bodies, their mass-weighted distribution places the global centre of mass at the selected coordinate origin. This arrangement is mechanically favourable because it reduces the likelihood that translational excitation will generate additional rotational effects solely as a consequence of an offset global mass centre. In practical terms, the chosen reference system allows the dynamic response to be interpreted with fewer coordinate-induced effects and provides a stable basis for subsequent vibration analysis.

The alignment of the principal inertia axes with the Cartesian coordinate directions further simplifies the mechanical description. Since all products of inertia are zero, the local inertia tensors are diagonal and no inertial coupling occurs between orthogonal rotational directions. This feature is particularly advantageous in a multibody simulation because the individual rotational contributions can be evaluated directly with respect to the selected coordinate axes. Additional tensor transformations are unnecessary, and any cross-axis response that may arise during simulation can be associated primarily with the mechanical constraints, imposed excitation or geometry of the interconnected bodies rather than with a misalignment of the inertia reference system.

The pronounced difference between longitudinal and transverse moments of inertia also provides insight into the directional mechanical behaviour of the assembly. The complete system possesses a substantially lower principal moment about the longitudinal axis than about either transverse axis. This ratio implies that the structure is less resistant to angular acceleration about the longitudinal direction, whereas rotational motion around the transverse axes requires a larger applied moment. Such anisotropy should be considered when vibration-induced perturbations are transmitted through the mechanical

chain, particularly if the excitation or constraint arrangement permits small rotational components to develop in addition to the prescribed translational motion.

Comparison between local and reference-frame inertia values demonstrates that spatial position has an influence comparable to, and in some cases greater than, the local inertia determined by component geometry. The dynamic platform illustrates this distinction most clearly. Its local transverse inertia is relatively small, yet its location at the largest longitudinal offset produces a considerable increase when the inertia is referred to the assembly reference frame. The resulting value is almost an order of magnitude greater than the local quantity. This behaviour follows directly from the parallel-axis contribution and confirms that the dynamic significance of a component depends not only on its mass and shape but also on its position within the overall mechanical arrangement.

A similar tendency is observed for the encoder housing and electric motor. The encoder housing has a moderate mass and local inertia, but its displacement from the assembly origin substantially raises its transverse reference-frame inertia. The electric motor, although positioned closer to the global centre, combines a comparatively large mass with a finite offset and therefore contributes appreciably to the global transverse inertia. These results show that components located at different positions can influence the system response in markedly different ways even when their local inertia values appear similar. The reference-frame representation is consequently essential when evaluating how each body contributes to the mechanical response of the complete assembly.

The optical rotary encoder itself exhibits relatively small principal moments of inertia owing to its limited mass and compact geometry. From a dynamic standpoint, this characteristic means that comparatively modest perturbing moments may produce measurable angular acceleration of the encoder body if the mounting arrangement does not provide sufficient constraint. At the same time, the encoder is not mechanically isolated. Its centre of mass is displaced from the assembly origin, and its response is mediated by the transmission mechanism, housing and supporting platforms. The encoder motion should therefore be regarded as an outcome of the complete structural configuration rather than as the response of an independent sensing element subjected directly to an imposed disturbance.

This interpretation is particularly important when the model is employed for vibration simulation. Mechanical excitation applied to the dynamic platform propagates through a connected multibody system whose response depends on the inertia and relative position of every rigid body. The resulting motion at the encoder is therefore governed by the collective mechanical arrangement. Such a formulation preserves the distinction between the source of vibration and the response of the sensing device,

which is an important requirement when the purpose of the simulation is to examine how structural transmission affects encoder behaviour.

The numerical comparison between the static and dynamic platforms further illustrates why component mass alone is an insufficient descriptor. The dynamic platform has a lower mass than the static platform, yet its reference-frame transverse inertia is considerably larger because its centre of mass lies much farther from the assembly origin. This contrast demonstrates that two bodies with broadly comparable masses may produce substantially different rotational contributions to the system merely because of their spatial placement. For model development, the combined use of mass, centre-of-mass coordinates and inertia is therefore more informative than reliance on any single physical property.

From a modelling perspective, the adopted formulation also provides a convenient basis for parametric modification. Changes in component geometry, material or position can be introduced through revised CAD-derived mass properties without restructuring the complete multibody model. Replacing the encoder, modifying the housing, changing the platform geometry or selecting a different material alters the corresponding masses, centre-of-mass coordinates and inertia tensors while the overall kinematic architecture remains unchanged. This feature is valuable for comparative studies in which several mechanical configurations must be assessed under equivalent excitation conditions.

The same approach can support design-oriented investigations. For example, moving a component closer to the assembly centre may reduce its reference-frame transverse inertia without altering its local inertia. Conversely, replacing a component with a denser material may increase both its local resistance to angular acceleration and its positional contribution to the global inertia. Such modifications can therefore be examined independently, allowing the mechanical influence of geometry, material and placement to be distinguished. The model can consequently serve not only as a representation of an existing assembly but also as a framework for evaluating alternative structural arrangements before experimental implementation.

Another relevant aspect is the numerical consistency observed between the component-level and assembly-level properties. The component masses, volumes and surface areas reproduce the corresponding quantities of the complete CAD assembly, indicating that the rigid-body decomposition preserves the physical content of the original geometry. This agreement strengthens confidence in the transfer of CAD-derived information into the multibody description. Since the subsequent dynamic equations depend directly on these properties, consistency at this stage is essential for avoiding discrepancies that could otherwise propagate into the simulated vibration response.

Nevertheless, the present analysis is primarily concerned with the inertial structure of the mechanical assembly rather than with a complete prediction of encoder measurement error. The results establish the mechanical basis required for such a simulation by defining how the individual rigid bodies contribute to the dynamic system. Further analysis may incorporate specific vibration amplitudes and frequencies, joint characteristics, contact conditions and experimentally identified damping parameters. These additions would permit the response obtained from the mass-properties-based model to be compared more directly with measured encoder behaviour under controlled vibration conditions.

Within these limits, the developed representation provides a coherent transition from CAD geometry to dynamic simulation. The main advantage lies in preserving the physical distinctions among the constituent bodies while maintaining a common assembly-level reference system. The resulting model retains component-specific mass and inertia characteristics, reflects their spatial arrangement and avoids unnecessary inertial coupling caused by coordinate misalignment. These properties make the formulation suitable for subsequent investigations of vibration transmission, structural sensitivity and encoder response under mechanically induced disturbances.

A further extension of the present framework may involve enriching the rigid-body representation with additional mechanical and measurement-related phenomena that are not explicitly considered at this stage. Particular attention may be directed towards experimentally identified damping, joint compliance, mounting stiffness and small structural clearances, since these effects can influence the transmission of vibration between the supporting elements and the encoder body. The excitation model may also be broadened from single-frequency harmonic motion to multi-frequency, transient or experimentally recorded vibration signals, thereby allowing the simulated mechanical response to represent a wider range of operating conditions.

Future development may additionally include a direct coupling between the calculated mechanical response and the metrological behaviour of the optical rotary encoder. In such an extension, vibration-induced translational and angular disturbances obtained from the multibody model could be related to variations in the measured angular position, rotational speed and corresponding measurement error. This would enable the mass-properties-based mechanical representation to form part of a more comprehensive simulation framework linking structural dynamics with encoder accuracy.

A subsequent stage may also incorporate systematic experimental validation. Parameters obtained from physical vibration tests could be used to refine the numerical model, while simulated displacements, accelerations and rotational deviations could be compared with measured quantities under equivalent excitation conditions. Such validation would permit assessment of model sensitivity and may identify which mass, inertia, positioning or constraint parameters exert the strongest influence on encoder response.

The formulation is likewise suitable for comparative and optimisation-oriented studies. Alternative encoder housings, supporting platforms, materials and component locations could be introduced through revised CAD-derived mass properties, allowing their influence to be examined without altering the fundamental multibody architecture. In this manner, the model could be progressively extended from a descriptive dynamic representation towards a design-support tool for selecting mechanical configurations that reduce vibration transmission and improve the operational stability of optical rotary encoder systems.

Overall, the results reinforce the premise that mass properties should be treated as an integrated modelling parameter set rather than as supplementary CAD output. The combination of mass, centre-of-mass position and inertia tensor defines how each rigid body participates in the dynamic behaviour of the assembly. Incorporating these quantities directly into the multibody model provides a mechanically traceable foundation for analysing the response of optical rotary encoders exposed to vibration and for examining how changes in structural configuration may affect that response.

CONCLUSION

A mass-properties-based representation provides a mechanically consistent basis for constructing dynamic models of optical rotary encoder assemblies subjected to external vibration. By retaining the physical characteristics of the constituent bodies and expressing them through quantities directly applicable to rigid multibody dynamics, the adopted formulation establishes a clear transition from CAD geometry to numerical simulation.

The analysed assembly has a total mass of 3.60419 kg and comprises an electric motor, transmission mechanism, optical rotary encoder, encoder housing, static platform and dynamic platform. The agreement between the aggregated component masses, volumes and surface areas and the corresponding values of the complete assembly confirms the internal consistency of the rigid-body decomposition. This correspondence indicates that the mechanical content of the CAD structure is preserved when the system is transferred into a component-level dynamic representation.

The centre-of-mass analysis further shows that all constituent bodies are distributed along the longitudinal axis, while the transverse coordinates remain zero. Their mass-weighted arrangement places the global centre of mass at the selected coordinate origin. Such a configuration provides a balanced mechanical reference and reduces the possibility that translational excitation will generate additional rotational effects purely as a consequence of coordinate offset.

The principal inertia axes of the complete assembly and of all constituent bodies coincide with the Cartesian coordinate directions. All products of

inertia are zero, and the resulting inertia tensors are therefore diagonal. This arrangement avoids coupling associated with non-zero products of inertia between the orthogonal principal directions.

A marked difference is observed between the longitudinal and transverse principal moments of inertia of the complete assembly. The longitudinal value is $0.00357 \text{ kg}\cdot\text{m}^2$, whereas both transverse values are $0.02338 \text{ kg}\cdot\text{m}^2$. This disparity indicates that the system offers substantially greater resistance to angular acceleration about the transverse directions than about its longitudinal axis. The result is important for subsequent vibration studies in which small rotational disturbances may accompany the imposed translational excitation.

Comparison between local and reference-frame inertia values demonstrates that the spatial position of an individual component can be as influential as its intrinsic mass and geometry. Components located farther from the assembly origin make considerably larger contributions to the transverse inertia when referred to the common reference frame. The dynamic platform provides the clearest example, with its local transverse inertia increasing from $0.00080 \text{ kg}\cdot\text{m}^2$ to $0.00879 \text{ kg}\cdot\text{m}^2$ as a consequence of its longitudinal offset.

These findings confirm that the mechanical behaviour of the encoder assembly cannot be characterised reliably through nominal encoder mass alone. The combined influence of component mass, centre-of-mass location and inertia tensor defines the dynamic role of each body within the interconnected structure. The surrounding transmission mechanism, housing, electric motor and supporting platforms therefore form an inseparable part of the vibration path that governs the mechanical response at the encoder location.

The developed formulation also provides a suitable basis for parametric modification and comparative assessment. Variations in component material, geometry or position can be introduced through revised CAD-derived mass properties without altering the fundamental multibody architecture. This enables alternative mechanical configurations to be examined under equivalent excitation conditions and creates a practical foundation for sensitivity-oriented or optimisation-based studies.

Further development may incorporate experimentally identified damping, mounting stiffness, joint compliance and other non-ideal mechanical effects that are not explicitly included in the present rigid-body formulation. The excitation description may likewise be extended beyond single-frequency harmonic motion to transient, broadband or experimentally recorded vibration signals. Such additions would permit a more detailed representation of operating conditions encountered in practical encoder installations.

A further step may involve coupling the mechanical response of the multibody system with the metrological behaviour of the optical rotary

encoder. Translational and angular disturbances obtained from the simulation could then be related directly to deviations in measured angular position, rotational speed and resulting measurement error. Experimental vibration testing may subsequently be used to refine the model parameters and assess the correspondence between simulated and measured encoder response.

Within the present scope, the results demonstrate that mass properties should be regarded as active modelling parameters rather than as supplementary CAD output. Their direct incorporation into the dynamic formulation preserves the inertial identity and spatial arrangement of the mechanical assembly and provides a reproducible basis for subsequent investigations of vibration transmission, encoder response and structurally induced measurement deviations.

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